CHAPTER 21 | MAGNETIC FORCES
AND MAGNETIC FIELDS

PROBLEMS

1. **SSM REASONING** The electron's acceleration is related to the net force $\Sigma F$ acting on it by Newton's second law: $a = \Sigma F/m$ (Equation 4.1), where $m$ is the electron's mass. Since we are ignoring the gravitational force, the net force is that caused by the magnetic force, whose magnitude is expressed by Equation 21.1 as $F = |q_0|vB\sin \theta$. Thus, the magnitude of the electron's acceleration can be written as $a = (|q_0|vB\sin \theta)/m$.

**SOLUTION** We note that $\theta = 90.0^\circ$, since the velocity of the electron is perpendicular to the magnetic field. The magnitude of the electron's charge is $1.60 \times 10^{-19}$ C, and the electron's mass is $9.11 \times 10^{-31}$ kg (see the inside of the front cover), so

$$a = \frac{|q_0|vB\sin \theta}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(2.1 \times 10^6 \text{ m/s})(1.6 \times 10^{-5} \text{ T})\sin 90.0^\circ}{9.11 \times 10^{-31} \text{ kg}} = 5.9 \times 10^{12} \text{ m/s}^2$$

2. **REASONING** The magnitude $B$ of the magnetic field is $B = \frac{F}{|q_0|v\sin \theta}$ (Equation 21.1), where $F$ is the magnitude of the magnetic force on the charge, whose magnitude is $|q_0|$ and whose velocity has a magnitude $v$ and makes an angle $\theta$ with the direction of the field. Both the proton in part a and the electron in part b have the same charge magnitude of $|q_0| = 1.60 \times 10^{-19}$ C. Therefore, the magnetic field has the same magnitude in both parts of the problem. However, the direction of the field is different for the proton and the electron. This is because the proton charge is positive, whereas the electron charge is negative. Finally, we note that the magnitude of the magnetic force is a maximum, which means that the velocity is perpendicular to the magnetic field, so that $\theta = 90.0^\circ$.

**SOLUTION**

a. Using Equation 21.1, we find that the magnitude of the magnetic field for the proton is

$$B = \frac{F}{|q_0|(v\sin \theta)} = \frac{8.0 \times 10^{-14} \text{ N}}{(1.60 \times 10^{-19} \text{ C})(4.5 \times 10^6 \text{ m/s})\sin 90.0^\circ} = 0.11 \text{ T}$$
Since the proton is traveling due east and the force points due south, we find from right hand rule no. 1 that the magnetic field points \[\text{upward, perpendicular to the earth's surface}.\]

b. For the electron, the magnitude of the field is the same as for the proton, since the two charges have the same magnitude. Thus, \[B = 0.11 \text{ T}\]. Since the electron is a negative charge, however, right-hand rule no. 1 reveals that the field direction is \[\text{downward, perpendicular to the earth's surface}.\]

3. **SSM REASONING** According to Equation 21.1, the magnitude of the magnetic force on a moving charge is \[F = |q_0| |vB\sin \theta|\]. Since the magnetic field points due north and the proton moves eastward, \[\theta = 90.0^\circ\]. Furthermore, since the magnetic force on the moving proton balances its weight, we have \[mg = |q_0| |vB\sin \theta|\], where \(m\) is the mass of the proton. This expression can be solved for the speed \(v\).

**SOLUTION** Solving for the speed \(v\), we have
\[
v = \frac{mg}{|q_0| |B\sin \theta|} = \frac{(1.67 \times 10^{-27} \text{ kg})(9.80 \text{ m/s}^2)}{(1.6 \times 10^{-19} \text{ C})(2.5 \times 10^{-5} \text{ T}) \sin 90.0^\circ} = 4.1 \times 10^{-3} \text{ m/s}
\]

4. **REASONING** The magnitude \(B\) of the magnetic field is given by \[B = \frac{F}{|q_0| |v\sin \theta|}\] (Equation 21.1), and we will apply this expression directly to obtain \(B\).

**SOLUTION** The charge \(q_0 = -8.3 \times 10^{-6} \text{ C}\) travels with a speed \(v = 7.4 \times 10^6 \text{ m/s}\) at an angle of \(\theta = 52^\circ\) with respect to a magnetic field of magnitude \(B\) and experiences a force of magnitude \(F = 5.4 \times 10^{-3} \text{ N}\). According to Equation 21.1, the field magnitude is
\[
B = \frac{F}{|q_0| |v\sin \theta|} = \frac{5.4 \times 10^{-3} \text{ N}}{|-8.3 \times 10^{-6} \text{ C}||(7.4 \times 10^6 \text{ m/s})\sin 52^\circ|} = 1.1 \times 10^{-4} \text{ T}
\]

Note in particular that it is only the magnitude \(|q_0|\) of the charge that appears in this calculation. The algebraic sign of the charge does not affect the result.

5. **REASONING** According to Equation 21.1, the magnetic force has a magnitude of \[F = |q| |vB\sin \theta|\], where \(|q|\) is the magnitude of the charge, \(B\) is the magnitude of the magnetic field, \(v\) is the speed, and \(\theta\) is the angle of the velocity with respect to the field. As \(\theta\) increases from \(0^\circ\) to \(90^\circ\), the force increases, so the angle must lie between \(25^\circ\) and \(90^\circ\).
SOLUTION Letting $\theta_1 = 25^\circ$ and $\theta_2$ be the desired angle, we can apply Equation 21.1 to both situations as follows:

$$F = \frac{|q|vB \sin \theta_1}{\text{Situation 1}} \quad \text{and} \quad 2F = \frac{|q|vB \sin \theta_2}{\text{Situation 2}}$$

Dividing the equation for situation 2 by the equation for situation 1 gives

$$\frac{2F}{F} = \frac{|q|vB \sin \theta_2}{|q|vB \sin \theta_1} \quad \text{or} \quad \sin \theta_2 = 2 \sin \theta_1 = 2 \sin 25^\circ = 0.85$$

$$\theta_2 = \sin^{-1}(0.85) = 58^\circ$$

6. REASONING A moving charge experiences no magnetic force when its velocity points in the direction of the magnetic field or in the direction opposite to the magnetic field. Thus, the magnetic field must point either in the direction of the $+x$ axis or in the direction of the $-x$ axis. If a moving charge experiences the maximum possible magnetic force when moving in a magnetic field, then the velocity must be perpendicular to the field. In other words, the angle $\theta$ that the charge's velocity makes with respect to the magnetic field is $\theta = 90^\circ$.

SOLUTION The magnitude $B$ of the magnetic field can be determined using Equation 21.1:

$$B = \frac{F}{|q|v \sin \theta} = \frac{0.48 \text{ N}}{(8.2 \times 10^{-6} \text{ C})(5.0 \times 10^5 \text{ m/s}) \sin 90^\circ} = 0.12 \text{ T}$$

In this calculation we use $\theta = 90^\circ$, because the 0.48-N force is the maximum possible force. Since the particle experiences no magnetic force when it moves along the $+x$ axis, we can conclude that the magnetic field points

either in the direction of the $+x$ axis or in the direction of the $-x$ axis.

7. REASONING The magnetic field applies the maximum magnetic force to the moving charge, because the motion is perpendicular to the field. This force is perpendicular to both the field and the velocity. The electric field applies an electric force to the charge that is in the same direction as the field, since the charge is positive. These two forces are shown in the drawing, and they are perpendicular to one another. Therefore, the magnitude of the net field can be obtained using the Pythagorean theorem.

SOLUTION According to Equation 21.1, the magnetic force has a magnitude of $F_{\text{magnetic}} = |q|vB \sin \theta$, where $|q|$ is the magnitude of the
charge, \( B \) is the magnitude of the magnetic field, \( v \) is the speed, and \( \theta = 90^\circ \) is the angle of the velocity with respect to the field. Thus, \( F_{\text{magnetic}} = |q|vB \). According to Equation 18.2, the electric force has a magnitude of \( F_{\text{electric}} = |q|E \). Using the Pythagorean theorem, we find the magnitude of the net force to be

\[
F = \sqrt{F_{\text{magnetic}}^2 + F_{\text{electric}}^2} = \sqrt{(|q|vB)^2 + (|q|E)^2} = |q|\sqrt{(vB)^2 + E^2}
\]

\[
= (1.8 \times 10^{-6} \text{ C}) \sqrt{[(3.1 \times 10^6 \text{ m/s})(1.2 \times 10^{-3} \text{ T})]^2 + (4.6 \times 10^3 \text{ N/C})^2} = 1.1 \times 10^{-2} \text{ N}
\]

8. **REASONING** According to Equation 21.1, the magnetic force has a magnitude of \( F = |q|vB \sin \theta \). The field \( B \) and the directional angle \( \theta \) are the same for each particle. Particle 1, however, travels faster than particle 2. By itself, a faster speed \( v \) would lead to a greater force magnitude \( F \). But the force on each particle is the same. Therefore, particle 1 must have a smaller charge to counteract the effect of its greater speed.

**SOLUTION** Applying Equation 21.1 to each particle, we have

\[
F = \frac{|q_1|v_1B \sin \theta}{\text{Particle 1}} \quad \text{and} \quad F = \frac{|q_2|v_2B \sin \theta}{\text{Particle 2}}
\]

Dividing the equation for particle 1 by the equation for particle 2 and remembering that \( v_1 = 3v_2 \) gives

\[
\frac{F}{F} = \frac{|q_1|v_1B \sin \theta}{v_2B \sin \theta} \quad \text{or} \quad 1 = \frac{|q_1|v_1}{|q_2|v_2} \quad \text{or} \quad \frac{|q_1|}{|q_2|} = \frac{v_2}{v_1} = \frac{3v_2}{v_2} = \frac{1}{3}
\]

9. **REASONING** The positive plate has a charge \( q \) and is moving downward with a speed \( v \) at right angles to a magnetic field of magnitude \( B \). The magnitude \( F \) of the magnetic force exerted on the positive plate is \( F = |q|vB \sin 90.0^\circ \). The charge on the positive plate is related to the magnitude \( E \) of the electric field that exists between the plates by (see Equation 18.4) \( |q| = \varepsilon_0AE \), where \( A \) is the area of the positive plate. Substituting this expression for \( |q| \) into \( F = |q|vB \sin 90.0^\circ \) gives the answer in terms of known quantities.

**SOLUTION**

\[
F = (\varepsilon_0AE)vB
\]

\[
= \left[ 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2) \right] \left( 7.5 \times 10^{-4} \text{ m}^2 \right) (170 \text{ N/C})(32 \text{ m/s})(3.6 \text{ T})
\]

\[
= 1.3 \times 10^{-10} \text{ N}
\]
An application of Right-Hand Rule No. 1 shows that the magnetic force is perpendicular to the plane of the page and directed out of the page, toward the reader.

10. **REASONING** The drawing on the left shows the directions of the two magnetic fields, as well as the velocity \( \mathbf{v} \) of the particle. Each component of the magnetic field is perpendicular to the velocity, so each exerts a magnetic force on the particle. The magnitude of the force is \( F = |q_0| v B \sin \theta \) (Equation 21.1), and the direction can be determined by using Right-Hand Rule No. 1 (RHR-1). The magnitude and direction of the net force can be found by using trigonometry.

![Diagram showing two magnetic fields and particle velocity](image)

**SOLUTION**

a. The magnitude \( F_1 \) of the magnetic force due to the 0.048-T magnetic field is

\[
F_1 = |q_0| v B_x \sin 90.0^\circ = \left(2.0 \times 10^{-5} \text{ C}\right) \left(4.2 \times 10^3 \text{ m/s}\right) (0.048 \text{ T}) = 4.0 \times 10^{-3} \text{ N}
\]

The magnitude \( F_2 \) of the magnetic force due to the 0.065-T magnetic field is

\[
F_2 = |q_0| v B_y \sin 90.0^\circ = \left(2.0 \times 10^{-5} \text{ C}\right) \left(4.2 \times 10^3 \text{ m/s}\right) (0.065 \text{ T}) = 5.5 \times 10^{-3} \text{ N}
\]

The directions of the forces are found using RHR-1, and they are indicated in the drawing on the right. Also shown is the net force \( \mathbf{F} \), as well as the angle \( \theta \) that it makes with respect to the +x axis. Since the forces are at right angles to each other, we can use the Pythagorean theorem to find the magnitude \( F \) of the net force:

\[
F = \sqrt{F_1^2 + F_2^2} = \sqrt{(4.0 \times 10^{-3} \text{ N})^2 + (5.5 \times 10^{-3} \text{ N})^2} = 6.8 \times 10^{-3} \text{ N}
\]

b. The angle \( \theta \) can be determined by using the inverse tangent function:

\[
\theta = \tan^{-1}\left(\frac{F_1}{F_2}\right) = \tan^{-1}\left(\frac{4.0 \times 10^{-3} \text{ N}}{5.5 \times 10^{-3} \text{ N}}\right) = 36^\circ
\]
11. **REASONING** The direction in which the electrons are deflected can be determined using Right-Hand Rule No. 1 and reversing the direction of the force (RHR-1 applies to positive charges, and electrons are negatively charged).

Each electron experiences an acceleration \( a \) given by Newton’s second law of motion, \( a = \frac{F}{m} \), where \( F \) is the net force and \( m \) is the mass of the electron. The only force acting on the electron is the magnetic force, \( F = q_0 |v| B \sin \theta \), so it is the net force. The speed \( v \) of the electron is related to its kinetic energy \( KE \) by the relation \( KE = \frac{1}{2}mv^2 \). Thus, we have enough information to find the acceleration.

**SOLUTION**

a. According to RHR-1, if you extend your right hand so that your fingers point along the direction of the magnetic field \( B \) and your thumb points in the direction of the velocity \( v \) of a positive charge, your palm will face in the direction of the force \( F \) on the positive charge.

For the electron in question, the fingers of the right hand should be oriented downward (direction of \( B \)) with the thumb pointing to the east (direction of \( v \)). The palm of the right hand points due north (the direction of \( F \) on a positive charge). Since the electron is negatively charged, it will be deflected [due south].

b. The acceleration of an electron is given by Newton’s second law, where the net force is the magnetic force. Thus,

\[
a = \frac{F}{m} = \frac{|q_0| v B \sin \theta}{m}
\]

Since the kinetic energy is \( KE = \frac{1}{2}mv^2 \), the speed of the electron is \( v = \sqrt{2(KE)/m} \). Thus, the acceleration of the electron is

\[
a = \frac{|q_0| v B \sin \theta}{m} = \frac{|q_0| \sqrt{\frac{2(KE)}{m}} B \sin \theta}{m}
\]

\[
= \left(1.60 \times 10^{-19} \text{ C}\right) \sqrt{\frac{2 \left(2.40 \times 10^{-15} \text{ J}\right)}{9.11 \times 10^{-31} \text{ kg}}} \left(2.00 \times 10^{-5} \text{ T}\right) \sin 90.0^\circ
\]

\[
= \frac{2.55 \times 10^{14} \text{ m/s}^2}{9.11 \times 10^{-31} \text{ kg}}
\]

12. **REASONING** Since \( e = 1.60 \times 10^{-19} \text{ C} \), we need to determine whether the charge has a magnitude of \( |q| = 1.60 \times 10^{-19} \text{ C} \) or \( |q| = 3.20 \times 10^{-19} \text{ C} \). We can do this by using

\[
r = \frac{mv}{|q| B} \quad \text{(Equation 21.2)},
\]

which gives the radius \( r \) of the circular path in terms of the mass
$m$ of the charged particle, the particle's speed $v$, and the magnitude $B$ of the magnetic field. This is possible since values are available for $r$, $m$, $v$, and $B$ in Equation 21.2.

**SOLUTION** Solving Equation 21.2 for $|q|$, we find that

$$|q| = \frac{mv}{Br} = \frac{(6.6 \times 10^{-27} \text{kg})(4.4 \times 10^5 \text{m/s})}{(0.75 \text{ T})(0.012 \text{ m})} = 3.2 \times 10^{-19} \text{ C}$$

This charge is $2e = 2(1.60 \times 10^{-19} \text{ C})$. We can see, then, that the charge of the ionized helium atom is $[+2e]$.

13. **SSM REASONING** The radius $r$ of the circular path is given by $r = \frac{mv}{|q|B}$ (Equation 21.2), where $m$ and $v$ are the mass and speed of the particle, respectively, $|q|$ is the magnitude of the charge, and $B$ is the magnitude of the magnetic field. This expression can be solved directly for $B$, since $r$, $m$, and $v$ are given and $q = +e$, where $e = 1.60 \times 10^{-19} \text{ C}$.

**SOLUTION** Solving Equation 21.2 for $B$ gives

$$B = \frac{|q|r}{mv} = \frac{(3.06 \times 10^{-25} \text{ kg})(7.2 \times 10^3 \text{ m/s})}{+1.60 \times 10^{-19} \text{ C}(0.10 \text{ m})} = 0.14 \text{ T}$$

14. **REASONING** The time $t$ that it takes the particle to complete one revolution is the time to travel a distance $d = 2\pi r$ equal to the circumference of a circle of radius $r$ at a speed $v$. From Equation 2.1, we know that speed is the ratio of distance to elapsed time ($v = d/t$), so the elapsed time is the ratio of distance to speed:

$$t = \frac{d}{v} = \frac{2\pi r}{v} \quad (1)$$

Because the particle follows a circular path that is perpendicular to the external magnetic field of magnitude $B$, the radius of the path is given by $r = \frac{mv}{|q|B}$ (Equation 21.2), where $m$ is the mass and $|q|$ is the magnitude of the charge of the particle. We will use Equation 21.2 to determine the speed of the particle, and then Equation (1) to find the time for one complete revolution.

**SOLUTION** Solving $r = \frac{mv}{|q|B}$ (Equation 21.2) for $v$ yields

$$v = \frac{|q|Br}{m} = \frac{|q|}{m} Br \quad (2)$$
In the last step of Equation (2), we have expressed the speed \( v \) explicitly in terms of the charge-to-mass ratio \( \frac{|q|}{m} \) of the particle. Substituting Equation (2) into Equation (1), we obtain

\[
I = \frac{2\pi r}{v} = \frac{2\pi}{\frac{|q|}{m} B} = \frac{2\pi}{(5.7 \times 10^8 \text{ C/kg})(0.72 \text{ T})} = 1.5 \times 10^{-8} \text{ s}
\]

15. **REASONING**

a. The drawing shows the velocity \( \mathbf{v} \) of the particle at the top of its path. The magnetic force \( \mathbf{F} \), which provides the centripetal force, must be directed toward the center of the circular path. Since the directions of \( \mathbf{v} \), \( \mathbf{F} \), and \( \mathbf{B} \) are known, we can use Right-Hand Rule No. 1 (RHR-1) to determine if the charge is positive or negative.

b. The radius of the circular path followed by a charged particle is given by Equation 21.2 as \( r = \frac{mv}{|q| B} \). The mass \( m \) of the particle can be obtained directly from this relation, since all other variables are known.

**SOLUTION**

a. If the particle were positively charged, an application of RHR-1 would show that the force would be directed straight up, opposite to that shown in the drawing. Thus, the charge on the particle must be **negative**.

b. Solving Equation 21.2 for the mass of the particle gives

\[
m = \frac{|q| B r}{v} = \frac{(8.2 \times 10^{-4} \text{ C})(0.48 \text{ T})(960 \text{ m})}{140 \text{ m/s}} = 2.7 \times 10^{-3} \text{ kg}
\]

16. **REASONING** Equation 21.2 gives the radius \( r \) of the circular path as \( r = \frac{mv}{(|q| B)} \), where \( m \), \( v \), and \( |q| \) are, respectively, the mass, speed, and charge magnitude of the particle, and \( B \) is the magnitude of the magnetic field. We wish the radius to be the same for both the proton and the electron. The speed \( v \) and the charge magnitude \( |q| \) are the same for the proton and the electron, but the mass of the electron is \( 9.11 \times 10^{-31} \text{ kg} \), while that of the proton is \( 1.67 \times 10^{-27} \text{ kg} \). Therefore, to offset the effect of the smaller electron mass \( m \) in Equation 21.2, the magnitude \( B \) of the field must be reduced for the electron.
SOLUTION Applying Equation 21.2 to the proton and the electron, both of which carry charges of the same magnitude \(|q| = e\), we obtain

\[
\frac{r_p}{eB_p} = \frac{m_p v_p}{eB_p} \quad \text{and} \quad \frac{r_e}{eB_e} = \frac{m_e v_e}{eB_e}
\]

Dividing the proton-equation by the electron-equation gives

\[
\frac{r_p}{r_e} = \frac{m_p v_p}{m_e v_e} \quad \text{or} \quad 1 = \frac{m_p B_e}{m_e B_p}
\]

Solving for \(B_e\), we obtain

\[
B_e = \frac{m_e B_p}{m_p} = \left(\frac{9.11 \times 10^{-31} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}}\right)(0.50 \text{ T}) = 2.7 \times 10^{-4} \text{ T}
\]

17. SSM REASONING As discussed in Section 21.4, the mass \(m\) of a singly-ionized particle that has been accelerated through a potential difference \(V\) and injected into a magnetic field of magnitude \(B\) is given by

\[
m = \left(\frac{er^2}{2V}\right)B^2
\]

where \(e = 1.60 \times 10^{-19} \text{ C}\) is the magnitude of the charge of an electron and \(r\) is the radius of the particle's path. If the beryllium-10 ions reach the same position in the detector as the beryllium-7 ions, both types of ions must have the same path radius \(r\). Additionally, the accelerating potential difference \(V\) is kept constant, so we see that the quantity \(\left(\frac{er^2}{2V}\right)\) in Equation (1) is the same for both types of ions.

SOLUTION All that differs between the two situations are the masses \((m_7, m_{10})\) of the ions and the magnitudes of the magnetic fields \((B_7, B_{10})\). Solving Equation (1) for the constant quantity \(\left(\frac{er^2}{2V}\right)\), we obtain

\[
\left(\frac{er^2}{2V}\right) = \frac{m_{10}}{B_{10}} = \frac{m_7}{B_7}
\]

(Same for both ions)
Solving Equation (2) for $B_{10}^2$, we find that

$$B_{10}^2 = B_7^2 \frac{m_{10}}{m_7} \quad \text{or} \quad B_{10} = B_7 \sqrt{\frac{m_{10}}{m_7}} = (0.283 \text{ T}) \frac{16.63 \times 10^{-27} \text{ kg}}{11.65 \times 10^{-27} \text{ kg}} = 0.338 \text{ T}$$

18. **REASONING** Section 21.4 discusses how a mass spectrometer determines the mass $m$ of an ion that has a charge of $+e$, where $e = 1.60 \times 10^{-19} \text{ C}$. This ion accelerates through a potential difference $V$ and follows a circular path (radius $r$) because of a magnetic field (magnitude $B$). The mass of the ion is $m = \left( \frac{e r^2}{2V} \right) B^2$. If the gold ions in this problem had a charge of $+e$, we could solve this expression directly for the radius $r$. However, the charge of the gold ions is $+2e$, so that before using the expression, we need to replace $e$ by $2e$.

**SOLUTION** Replacing $e$ by $2e$ in the expression from Section 21.4 gives the mass as

$$m = \left( \frac{2e r^2}{2V} \right) B^2.$$ Solving this equation for the radius $r$, we find that

$$r = \frac{\sqrt{2mV}}{2eB^2} = \frac{\sqrt{2 \left( 3.27 \times 10^{-25} \text{ kg} \right) \left( 1.00 \times 10^3 \text{ V} \right)}}{2 \left( 1.60 \times 10^{-19} \text{ C} \right) \left( 0.500 \text{ T} \right)^2} = 0.0904 \text{ m}$$

19. **REASONING** The speed of the $\alpha$-particle can be obtained by applying the principle of conservation of energy, recognizing that the total energy is the sum of the particle’s kinetic energy and electric potential energy, the gravitational potential energy being negligible in comparison. Once the speed is known, Equation 21.1 can be used to obtain the magnitude of the magnetic force that acts on the particle. Lastly, the radius of its circular path can be obtained directly from Equation 21.2.

**SOLUTION**

a. Using A and B to denote the initial positions, respectively, the principle of conservation of energy can be written as follows:

$$\frac{1}{2} m_B v_B^2 + \text{EPE}_B = \frac{1}{2} m_A v_A^2 + \text{EPE}_A$$

Using Equation 19.3 to express the electric potential energy of the charge $q_0$ as $\text{EPE} = q_0V$, where $V$ is the electric potential, we find from Equation (1) that

$$\frac{1}{2} m_B v_B^2 + q_0 V_B = \frac{1}{2} m_A v_A^2 + q_0 V_A$$

Since the particle starts from rest, we have that $v_A = 0 \text{ m/s}$, and Equation (2) indicates that
\[ v_B = \sqrt{\frac{2q_0 (V_A - V_B)}{m}} = \sqrt{\frac{2 \left[ 2 \left( +1.60 \times 10^{-19} \text{ C} \right) \right]}{6.64 \times 10^{-27} \text{ kg}}} \left( 1.20 \times 10^6 \text{ V} \right) = 1.08 \times 10^7 \text{ m/s} \]

b. According to Equation 21.1, the magnitude of the magnetic force that acts on the particle is

\[ F = q_0 v_B B \sin \theta = 2 \left( 1.60 \times 10^{-19} \text{ C} \right) \left( 1.08 \times 10^7 \text{ m/s} \right) \left( 2.20 \text{ T} \right) \sin 90.0^\circ = 7.60 \times 10^{-12} \text{ N} \]

where \( \theta = 90.0^\circ \), since the particle travels perpendicular to the field at all times.

c. According to Equation 21.2, the radius of the circular path on which the particle travels is

\[ r = \frac{mv_B}{q_0 B} = \frac{6.64 \times 10^{-27} \text{ kg} \left( 1.08 \times 10^7 \text{ m/s} \right)}{2 \left( 1.60 \times 10^{-19} \text{ C} \right) \left( 2.20 \text{ T} \right)} = 0.102 \text{ m} \]

20. **REASONING** Equation 21.2 gives the radius \( r \) of the circular path as \( r = \frac{mv}{|q|B} \), where \( m, v, \) and \( |q| \) are, respectively, the mass, speed, and charge magnitude of the particle, and \( B \) is the magnitude of the magnetic field.

We can determine the speed of each particle by employing the principle of conservation of energy. The electric potential energy lost as the particles accelerate is converted into kinetic energy. Equation 19.4 indicates that the electric potential energy lost is \( |q|V \), where \( |q| \) is the magnitude of the charge and \( V \) is the electric potential difference. Since \( q \) and \( V \) are the same for each particle, each loses the same amount of potential energy. Energy conservation, then, dictates that each gains the same amount of kinetic energy. Since each particle starts from rest, each enters the magnetic field with the same amount of kinetic energy.

**SOLUTION** According to Equation 21.2, \( r = \frac{mv}{|q|B} \). To determine the speed \( v \) with which each particle enters the field, we use Equation 19.4 and the energy-conservation principle as follows:

\[
\frac{|q|V}{\text{Electric potential energy lost}} = \frac{1}{2} \frac{mv^2}{\text{Kinetic energy gained}} \quad \text{or} \quad v = \sqrt{\frac{2|q|V}{m}}
\]

Substituting this result into Equation 21.2 gives the radius of the circular motion:

\[
r = \frac{mv}{|q|B} = \frac{m}{|q|B} \sqrt{\frac{2|q|V}{m}} = \frac{1}{B} \sqrt{\frac{2mV}{|q|}}
\]

Applying this result to each particle, we obtain
Dividing \( r_2 \) by \( r_1 \) gives

\[
\frac{r_2}{r_1} = \frac{1}{B} \sqrt{\frac{2m_2V}{|q|}} = \frac{m_2}{B \sqrt{m_1}}
\]

\[
r_2 = r_1 \sqrt{\frac{m_2}{m_1}} = (12 \text{ cm}) \sqrt{\frac{5.9 \times 10^{-8} \text{ kg}}{2.3 \times 10^{-8} \text{ kg}}} = 19 \text{ cm}
\]

**21. REASONING** The drawing shows the velocity \( \mathbf{v} \) of the carbon atoms as they enter the magnetic field \( \mathbf{B} \). The diameter of the circular path followed by the carbon-12 atoms is labeled as \( 2r_{12} \), and that of the carbon-13 atoms as \( 2r_{13} \), where \( r \) denotes the radius of the path. The radius is given by Equation 21.2 as \( r = \frac{mv}{|q|B} \), where \( q \) is the charge on the ion \( (q = +e) \). The difference \( \Delta d \) in the diameters is \( \Delta d = 2r_{13} - 2r_{12} \) (see the drawing).

**SOLUTION** The spatial separation between the two isotopes after they have traveled though a half-circle is

\[
\Delta d = 2r_{13} - 2r_{12} = 2 \left( \frac{m_{13}v}{eB} \right) - 2 \left( \frac{m_{12}v}{eB} \right) = 2v \left( \frac{m_{13}}{eB} - \frac{m_{12}}{eB} \right)
\]

\[
= \frac{2 \left( 6.667 \times 10^5 \text{ m/s} \right)}{(1.60 \times 10^{-19} \text{ C})(0.8500 \text{ T})} \left( 21.59 \times 10^{-27} \text{ kg} - 19.93 \times 10^{-27} \text{ kg} \right) = 1.63 \times 10^{-2} \text{ m}
\]

**22. REASONING** The radius of the circular path is given by Equation 21.2 as \( r = \frac{mv}{|q|B} \), where \( m \) is the mass of the species, \( v \) is the speed, \( |q| \) is the magnitude of the charge, and \( B \) is the magnitude of the magnetic field. To use this expression, we must know something about the speed. Information about the speed can be obtained by applying the conservation of energy principle. The electric potential energy lost as a charged particle "falls" from a higher to a lower electric potential is gained by the particle as kinetic energy.
**SOLUTION** For an electric potential difference $V$ and a charge $q$, the electric potential energy lost is $|q|V$, according to Equation 19.4. The kinetic energy gained is $\frac{1}{2}mv^2$. Thus, energy conservation dictates that

$$|q|V = \frac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{\frac{2|q|V}{m}}$$

Substituting this result into Equation 21.2 for the radius gives

$$r = \frac{mv}{|q|B} = \frac{m}{|q|B} \sqrt{\frac{2|q|V}{m}} = \frac{1}{B} \sqrt{\frac{2mV}{|q|}}$$

Using $e$ to denote the magnitude of the charge on an electron, we note that the charge for species $X^+$ is $+e$, while the charge for species $X^{2+}$ is $+2e$. With this in mind, we find for the ratio of the radii that

$$\frac{r_1}{r_2} = \frac{1}{B} \sqrt{\frac{2mV}{e}} = \sqrt{2} = \frac{1.41}{1}$$

23. **SSM REASONING** When the proton moves in the magnetic field, its trajectory is a circular path. The proton will just miss the opposite plate if the distance between the plates is equal to the radius of the path. The radius is given by Equation 21.2 as $r = \frac{mv}{(|q|B)}$. This relation can be used to find the magnitude $B$ of the magnetic field, since values for all the other variables are known.

**SOLUTION** Solving the relation $r = \frac{mv}{(|q|B)}$ for the magnitude of the magnetic field, and realizing that the radius is equal to the plate separation, we find that

$$B = \frac{mv}{|q|r} = \frac{(1.67 \times 10^{-27} \text{ kg}) (3.5 \times 10^6 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C}) (0.23 \text{ m})} = 0.16 \text{ T}$$

The values for the mass and the magnitude of the charge (which is the same as that of the electron) have been taken from the inside of the front cover.
24. **REASONING** The magnitude $F_B$ of the magnetic force acting on the particle is related to its speed $v$ by $F_B = q_0 |v| B \sin \theta$ (Equation 21.1), where $B$ is the magnitude of the magnetic field, $q_0$ is the particle's charge, and $\theta$ is the angle between the magnetic field $B$ and the particle's velocity $v$. As the drawing shows, the vector $v$ (east, to the right) is perpendicular to the vector $B$ (south, out of the page). Therefore, $\theta = 90^\circ$, and Equation 21.1 becomes

$$F_B = |q_0|vB \sin 90^\circ = |q_0|vB$$  \hspace{1cm} (1)

In addition to the magnetic force, there is also an electric force of magnitude $F_E$ acting on the particle. This force magnitude does not depend upon the speed $v$ of the particle, as we see from $F_E = |q_0|E$ (Equation 18.2). The particle is positively charged, so the electric force acting on it points upward in the same direction as the electric field. By Right-Hand Rule No.1, the magnetic force acting on the positively charged particle points down, and is therefore opposite to the electric force. The net force on the particle points upward, so we conclude that the electric force is greater than the magnetic force. Thus, the magnitude $F$ of the net force acting on the particle is equal to the magnitude of the electric force minus the magnitude of the magnetic force:

$$F = F_E - F_B$$  \hspace{1cm} (2)

We also note that particles traveling at a speed $v_0 = 6.50 \times 10^3$ m/s experience no net force. Therefore, $F_E = F_B$ for particles moving at the speed $v_0$.

**SOLUTION** Substituting Equation (1) and $F_E = |q_0|E$ (Equation 18.2) into Equation (2) yields

$$F = |q_0|E - |q_0|vB$$  \hspace{1cm} (3)

The magnetic field magnitude $B$ is not given, but, as noted above, for particles with speed $v_0 = 6.50 \times 10^3$ m/s, the magnetic force of Equation (1), $F_B = |q_0|v_0 B$, is equal to the electric force $F_E = |q_0|E$ (Equation 18.2). Therefore, we have that

$$|q_0|v_0 B = |q_0|E \quad \text{or} \quad B = \frac{E}{v_0}$$  \hspace{1cm} (4)

Substituting Equation (4) into Equation (3) yields

$$F = |q_0|E - |q_0|vB = |q_0|E - \frac{|q_0|vE}{v_0}$$  \hspace{1cm} (5)
Solving Equation (5) for \( v \), we obtain
\[
\frac{q_0 v E}{v_0} = |q_0| E - F \quad \text{or} \quad \frac{v}{v_0} = 1 - \frac{F}{|q_0| E} \quad \text{or} \quad v = v_0 \left( 1 - \frac{F}{|q_0| E} \right)
\]  
(6)

Substituting the given values into Equation (6), we find that
\[
v = v_0 \left( 1 - \frac{F}{|q_0| E} \right) = (6.50 \times 10^3 \text{ m/s}) \left[ 1 - \frac{1.90 \times 10^{-9} \text{ N}}{4.00 \times 10^{-12} \text{ C}(2470 \text{ N/C})} \right] = 5.25 \times 10^3 \text{ m/s}
\]

25. **SSM REASONING** The particle travels in a semicircular path of radius \( r \), where \( r \) is given by Equation 21.2 \( r = \frac{mv}{|q| B} \). The time spent by the particle in the magnetic field is given by \( t = s/v \), where \( s \) is the distance traveled by the particle and \( v \) is its speed. The distance \( s \) is equal to one-half the circumference of a circle \( (s = \pi r) \).

**SOLUTION** We find that
\[
t = \frac{s}{v} = \frac{\pi r}{v} = \frac{\pi}{v} \frac{mv}{|q| B} = \frac{\pi m}{|q| B} \frac{(6.0 \times 10^{-8} \text{ kg})}{(7.2 \times 10^{-6} \text{ C})(3.0 \text{ T})} = 8.7 \times 10^{-3} \text{ s}
\]

26. **REASONING** When the electron travels perpendicular to a magnetic field, its path is a circle. The radius of the circle is given by Equation 21.2 as \( r = \frac{mv}{(|q| B)} \). All the variables in this relation are known, except the speed \( v \). However, the speed is related to the electron’s kinetic energy \( KE \) by \( KE = \frac{1}{2} mv^2 \) (Equation 6.2). By combining these two relations, we will be able to find the radius of the path.

**SOLUTION** Solving the relation \( KE = \frac{1}{2} mv^2 \) for the speed and substituting the result into \( r = \frac{mv}{(|q| B)} \) give
\[
r = \frac{mv}{|q| B} = \frac{\sqrt{2(KE)}}{m} = \frac{\sqrt{2m(KE)}}{|q| B} = \frac{\sqrt{2\left(9.11 \times 10^{-31} \text{ kg}\right)\left(2.0 \times 10^{-17} \text{ J}\right)}}{(1.60 \times 10^{-19} \text{ C})(5.3 \times 10^{-5} \text{ T})} = 0.71 \text{ m}
\]

Values for the mass and charge of the electron have been taken from the inside of the front cover.
27. **REASONING**

a. When the particle moves in the magnetic field, its path is circular. To keep the particle moving on a circular path, it must experience a centripetal force, the magnitude of which is given by Equation 5.3 as \( F_c = \frac{mv^2}{r} \). In the present situation, the magnetic force \( F \) furnishes the centripetal force, so \( F_c = F \). The mass \( m \) and speed \( v \) of the particle are known, but the radius \( r \) of the path is not. However, the particle travels at a constant speed, so in a time \( t \) the distance \( s \) it travels is \( s = vt \). But the distance is one-quarter of the circumference \( (2\pi r) \) of a circle, so \( s = \frac{1}{4}(2\pi r) \). By combining these three relations, we can determine the magnitude of the magnetic force.

b. The magnitude of the magnetic force is given by Equation 21.1 as \( F = |q|vB\sin \theta \). Since \( F, v, B, \) and \( \theta \) are known, this relation can be used to determine the magnitude \( |q| \) of the charge.

**SOLUTION**

a. The magnetic force, which provides the centripetal force, is \( F = \frac{mv^2}{r} \). Solving the relation \( s = \frac{1}{4}(2\pi r) \) for the radius and substituting \( s = vt \) into the result gives

\[
r = \frac{2s}{\pi} = \frac{2(\frac{vt}{\pi})}{\pi}
\]

Using this expression for \( r \) in Equation 5.3, we find that the magnitude of the magnetic force is

\[
F = \frac{mv^2}{r} = \frac{\frac{mv^2}{2t}}{\frac{2(vt)}{\pi}} = \frac{\pi \left( \frac{7.2 \times 10^{-8} \text{ kg}}{2(2.2 \times 10^{-3} \text{ s})} \right) (85 \text{ m/s})}{2} = 4.4 \times 10^{-3} \text{ N}
\]

b. Solving the relation \( F = |q|vB\sin \theta \) for the magnitude \( |q| \) of the charge and noting that \( \theta = 90.0^\circ \) (since the velocity of the particle is perpendicular to the magnetic field), we find that

\[
|q| = \frac{F}{vB\sin 90^\circ} = \frac{4.4 \times 10^{-3} \text{ N}}{(85 \text{ m/s})(0.31 \text{ T})\sin 90^\circ} = 1.7 \times 10^{-4} \text{ C}
\]

28. **REASONING AND SOLUTION** The magnitudes of the magnetic and electric forces must be equal. Therefore,

\[
F_B = F_E \quad \text{or} \quad |q|vB = |q|E
\]

This relation can be solved to give the speed of the particle, \( v = E/B \). We also know that when the electric field is turned off, the particle travels in a circular path of radius \( r = \frac{mv}{|q|B} \). Substituting \( v = E/B \) into this equation and solving for \( |q|/m \) gives...
\[
\frac{|q|}{m} = \frac{E}{rB^2} = \frac{3.80 \times 10^3 \text{ N/C}}{(4.30 \times 10^{-2} \text{ m})(0.360 \text{ T})^2} = 6.8 \times 10^5 \text{ C/kg}
\]

29. **REASONING AND SOLUTION** The following drawings show the two circular paths leading to the target T when the proton is projected from the origin O. In each case, the center of the circle is at C. Since the target is located at \(x = -0.10 \text{ m}\) and \(y = -0.10 \text{ m}\), the radius of each circle is \(r = 0.10 \text{ m}\). The speed with which the proton is projected can be obtained from Equation 21.2, if we remember that the charge and mass of a proton are \(q = +1.60 \times 10^{-19} \text{ C}\) and \(m = 1.67 \times 10^{-27} \text{ kg}\), respectively:

\[
v = \frac{r |q| B}{m} = \frac{(0.10 \text{ m})(1.60 \times 10^{-19} \text{ C})(0.010 \text{ T})}{1.67 \times 10^{-27} \text{ kg}} = 9.6 \times 10^4 \text{ m/s}
\]

30. **REASONING** A magnetic field exerts no force on a current-carrying wire that is directed along the same direction as the field. This follows directly from \(F = ILB \sin \theta\) (Equation 21.3), which gives the magnitude \(F\) of the magnetic force that acts on a wire of length \(L\) that is directed at an angle \(\theta\) with respect to a magnetic field of magnitude \(B\) and carries a current \(I\). When \(\theta = 0^\circ\) or \(180^\circ\), \(F = 0\) N. Therefore, we need only apply Equation 21.3 to the horizontal component of the earth's magnetic field in this problem. The direction of the magnetic force can be determined with the aid of RHR-1 (fingers point in direction of the field, thumb points in the direction of the current, palm faces in the direction of the magnetic force).

**SOLUTION** According to Equation 21.3, the magnitude of the magnetic force exerted on the wire by the horizontal component of the earth's field is

\[
F = ILB \sin \theta = (28 \text{ A})(6.0 \text{ m})(1.8 \times 10^{-5} \text{ T}) \sin 90.0^\circ = 3.0 \times 10^{-3} \text{ N}
\]
Note that $\theta = 90.0^\circ$ because the field component points toward the geographic north and the current is directed perpendicularly into the ground. The application of RHR-1 (fingers point due north, thumb points perpendicularly into the ground, palm faces due east) reveals that the direction of the magnetic force is due east.

31. **SSM REASONING** The magnitude $F$ of the magnetic force experienced by the wire is given by $F = ILB \sin \theta$ (Equation 21.3), where $I$ is the current, $L$ is the length of the wire, $B$ is the magnitude of the earth’s magnetic field, and $\theta$ is the angle between the direction of the current and the magnetic field. Since all the variables are known except $B$, we can use this relation to find its value.

**SOLUTION** Solving $F = ILB \sin \theta$ for the magnitude of the magnetic field, we have

$$B = \frac{F}{IL \sin \theta} = \frac{0.15 \text{ N}}{(75 \text{ A})(45 \text{ m}) \sin 60.0^\circ} = 5.1 \times 10^{-5} \text{ T}$$

32. **REASONING** The magnitude $F$ of the force on a current $I$ is given by Equation 21.3 as $F = ILB \sin \theta$ (Equation 21.3), where $L$ is the length of the wire and $\theta$ is the angle between the wire and a magnetic field that has a magnitude $B$. We can apply this equation to both situations. The key to the solution is the fact that $L$, $B$, and $\theta$, although unknown, have the same values in both cases. This fact will allow us to eliminate them algebraically from the calculation of the unknown current.

**SOLUTION** Initially we have $F_1 = 0.030 \text{ N}$ and $I_1 = 2.7 \text{ A}$. In the second case, we have $F_2 = 0.047 \text{ N}$ and $I_2$. Applying this Equation 21.3 to both situations we have

$$F_1 = I_1LB \sin \theta \quad \text{and} \quad F_2 = I_2LB \sin \theta$$

Dividing the right-hand equation by the left-hand equation gives

$$\frac{F_2}{F_1} = \frac{I_2LB \sin \theta}{I_1LB \sin \theta} = \frac{I_2}{I_1} \quad \text{or} \quad I_2 = I_1 \left( \frac{F_2}{F_1} \right) = (2.7 \text{ A}) \left( \frac{0.047 \text{ N}}{0.030 \text{ N}} \right) = 4.2 \text{ A}$$

33. **REASONING** The magnitude $B$ of the external magnetic field is proportional to the magnitude $F$ of the magnetic force exerted on the wire, according to $F = ILB \sin \theta$ (Equation 21.3), where $L$ is the length of the wire, $I$ is the current it carries, and $\theta$ is the angle between the directions of the current and the magnetic field. When the wire is horizontal, the magnetic force is zero, indicating that $\sin \theta = 0$. The only angles $\theta$ for which this holds are $\theta = 0^\circ$ and $\theta = 180^\circ$. Therefore, the external magnetic field must be horizontal, and when the wire is tilted upwards at an angle of $19^\circ$, the angle between the directions of the current and the magnetic field must be $\theta = 19^\circ$. 
**SOLUTION** Solving $F = ILB\sin \theta$ (Equation 21.3) for $B$ yields

$$B = \frac{F}{IL\sin \theta} = \frac{4.4 \times 10^{-3} \text{ N}}{(7.5 \text{ A})(0.53 \text{ m})\sin 19^\circ} = \frac{3.4 \times 10^{-3} \text{ T}}{\text{N}}$$

34. **REASONING** We begin by noting that segments AB and BC are both perpendicular to the magnetic field. Therefore, they experience magnetic forces. However, segment CD is parallel to the field. As a result no magnetic force acts on it. According to Equation 21.3, the magnitude $F$ of the magnetic force on a current $I$ is $F = ILB\sin \theta$, where $L$ is the length of the wire segment and $\theta$ is the angle that the current makes with respect to the magnetic field. For both segments AB and BC the value of the current is the same and the value of $\theta$ is $90^\circ$. The length of segment AB is greater, however. Because of its greater length, segment AB experiences the greater force.

**SOLUTION** Using $F = ILB\sin \theta$ (Equation 21.3), we find that the magnitudes of the magnetic forces acting on the segments are:

- **Segment AB**
  $$F = ILB\sin \theta = 2.8 \text{ A} \cdot 0.1 \text{ m} \cdot 0.26 \text{ T} \cdot \sin 90^\circ = 0.80 \text{ N}$$

- **Segment BC**
  $$F = ILB\sin \theta = 2.8 \text{ A} \cdot 0.55 \text{ m} \cdot 0.26 \text{ T} \cdot \sin 90^\circ = 0.40 \text{ N}$$

- **Segment CD**
  $$F = ILB\sin \theta = 2.8 \text{ A} \cdot 0.55 \text{ m} \cdot 0.26 \text{ T} \cdot \sin 0^\circ = 0 \text{ N}$$

35. **SSM REASONING** According to Equation 21.3, the magnetic force has a magnitude of $F = ILB\sin \theta$, where $I$ is the current, $B$ is the magnitude of the magnetic field, $L$ is the length of the wire, and $\theta = 90^\circ$ is the angle of the wire with respect to the field.

**SOLUTION** Using Equation 21.3, we find that

$$L = \frac{F}{IB\sin \theta} = \frac{7.1 \times 10^{-5} \text{ N}}{0.66 \text{ A} \cdot 0.7 \times 10^{-5} \text{ T} \cdot \sin 58^\circ} = \frac{2.7 \text{ m}}{\text{m}}$$

36. **REASONING** Each wire experiences a force due to the magnetic field. The magnitude of the force is given by $F = ILB\sin \theta$ (Equation 21.3), where $I$ is the current, $L$ is the length of the wire, $B$ is the magnitude of the magnetic field, and $\theta$ is the angle between the direction of the current and the magnetic field. Since the currents in the two wires are in opposite directions, the magnetic force acting on one wire is opposite to that acting on the other. Thus, the net force acting on the two-wire unit is the difference between the magnitudes of the forces acting on each wire.

**SOLUTION** The length $L$ of each wire, the magnetic field $B$, and the angle $\theta$ are the same for both wires. Denoting the current in one of the wires as $I_1 = 7.00 \text{ A}$ and the current in the other as $I$, the magnitude $F_{\text{net}}$ of the net magnetic force acting on the two-wire unit is
\[ F_{\text{net}} = I_1LB \sin \theta - ILB \sin \theta = (I_1 - I)LB \sin \theta \]

Solving for the unknown current \( I \) gives

\[ I = I_1 - \frac{F_{\text{net}}}{LB \sin \theta} = 7.00 \text{ A} - \frac{3.13 \text{ N}}{(2.40 \text{ m})(0.360 \text{ T}) \sin 65.0^\circ} = 3.00 \text{ A} \]

37. **REASONING** The magnitude of the magnetic force exerted on a long straight wire is given by Equation 21.3 as \( F = ILB \sin \theta \). The direction of the magnetic force is predicted by Right-Hand Rule No. 1. The net force on the triangular loop is the vector sum of the forces on the three sides.

**SOLUTION**

a. The direction of the current in side \( AB \) is opposite to the direction of the magnetic field, so the angle \( \theta \) between them is \( \theta = 180^\circ \). The magnitude of the magnetic force is

\[ F_{AB} = ILB \sin \theta = ILB \sin 180^\circ = 0 \text{ N} \]

For the side \( BC \), the angle is \( \theta = 55.0^\circ \), and the length of the side is

\[ L = \frac{2.00 \text{ m}}{\cos 55.0^\circ} = 3.49 \text{ m} \]

The magnetic force is

\[ F_{BC} = ILB \sin \theta = 9.70 \text{ A} \cdot 3.49 \text{ m} \cdot 0.80 \text{ T} \cdot \sin 55.0^\circ = 24.2 \text{ N} \]

An application of Right-Hand No. 1 shows that the magnetic force on side \( BC \) is directed perpendicularly out of the paper, toward the reader.

For the side \( AC \), the angle is \( \theta = 90.0^\circ \). We see that the length of the side is

\[ L = (2.00 \text{ m}) \tan 55.0^\circ = 2.86 \text{ m} \]

The magnetic force is

\[ F_{AC} = ILB \sin \theta = 9.70 \text{ A} \cdot 2.86 \text{ m} \cdot 0.80 \text{ T} \cdot \sin 90.0^\circ = 24.2 \text{ N} \]

An application of Right-Hand No. 1 shows that the magnetic force on side \( AC \) is directed perpendicularly into the paper, away from the reader.

b. The net force is the vector sum of the forces on the three sides. Taking the positive direction as being out of the paper, the net force is

\[ \sum F = 0 \text{ N} + 24.2 \text{ N} + 24.2 \text{ N} = 0 \text{ N} \]
38. **REASONING AND SOLUTION**

a. From Right-Hand Rule No. 1, if we extend the right hand so that the fingers point in the direction of the magnetic field, and the thumb points in the direction of the current, the palm of the hand faces the direction of the magnetic force on the current.

The springs will stretch when the magnetic force exerted on the copper rod is downward, toward the bottom of the page. Therefore, if you extend your right hand with your fingers pointing out of the page and the palm of your hand facing the bottom of the page, your thumb points left-to-right along the copper rod. Thus, the current flows **left-to-right** in the copper rod.

b. The downward magnetic force exerted on the copper rod is, according to Equation 21.3

\[ F = I L B \sin \theta = (12 \text{ A})(0.85 \text{ m})(0.16 \text{ T}) \sin 90.0^\circ = 1.6 \text{ N} \]

According to Equation 10.1, the force \( F_x^{\text{Applied}} \) required to stretch each spring is \( F_x^{\text{Applied}} = kx \), where \( k \) is the spring constant. Since there are two springs, we know that the magnetic force \( F \) exerted on the current must equal \( 2F_x^{\text{Applied}} \), so that \( F = 2F_x^{\text{Applied}} = 2kx \). Solving for \( x \), we find that

\[
x = \frac{F}{2k} = \frac{1.6 \text{ N}}{2(75 \text{ N/m})} = 1.1 \times 10^{-2} \text{ m}
\]

39. **SSM REASONING** Since the rod does not rotate about the axis at \( P \), the net torque relative to that axis must be zero; \( \Sigma \tau = 0 \) (Equation 9.2). There are two torques that must be considered, one due to the magnetic force and another due to the weight of the rod. We consider both of these to act at the rod's center of gravity, which is at the geometrical center of the rod (length = \( L \)), because the rod is uniform. According to Right-Hand Rule No. 1, the magnetic force acts perpendicular to the rod and is directed up and to the left in the drawing. Therefore, the magnetic torque is a counterclockwise (positive) torque. Equation 21.3 gives the magnitude \( F \) of the magnetic force as \( F = I L B \sin 90.0^\circ \), since the current is perpendicular to the magnetic field. The weight is \( mg \) and acts downward, producing a clockwise (negative) torque. The magnitude of each torque is the magnitude of the force times the lever arm (Equation 9.1). Thus, we have for the torques:

\[
\tau_{\text{magnetic}} = \frac{(L B)(L/2)}{\text{force lever arm}} \quad \text{and} \quad \tau_{\text{weight}} = -\frac{(mg)(L/2 \cos \theta)}{\text{force lever arm}}
\]

Setting the sum of these torques equal to zero will enable us to find the angle \( \theta \) that the rod makes with the ground.
SOLUTION Setting the sum of the torques equal to zero gives \( \Sigma \tau = \tau_{\text{magnetic}} + \tau_{\text{weight}} = 0 \), and we have

\[
+ (ILB)(L/2) - (mg)[(L/2) \cos \theta] = 0 \quad \text{or} \quad \cos \theta = \frac{ILB}{mg}
\]

\[
\theta = \cos^{-1} \left[ \frac{(4.1 \text{ A})(0.45 \text{ m})(0.36 \text{ T})}{(0.094 \text{ kg})(9.80 \text{ m/s}^2)} \right] = 44^\circ
\]

40. REASONING
There are four forces that act on the wire: the magnetic force (magnitude \( F \)), the weight \( mg \) of the wire, and the tension in each of the two strings (magnitude \( T \) in each string). Since there are two strings, the following drawing shows the total tension as \( 2T \). The magnitude \( F \) of the magnetic force is given by \( F = ILB \sin \theta \) (Equation 21.3), where \( I \) is the current, \( L \) is the length of the wire, \( B \) is the magnitude of the magnetic field, and \( \theta \) is the angle between the wire and the magnetic field. In this problem \( \theta = 90.0^\circ \).

The direction of the magnetic force is given by Right-Hand Rule No. 1 (see Section 21.5). The drawing shows an end view of the wire, where it can be seen that the magnetic force (magnitude \( = F \)) points to the right, in the +x direction.

![Diagram](image)

In order for the wire to be in equilibrium, the net force \( \Sigma F_x \) in the \( x \)-direction must be zero, and the net force \( \Sigma F_y \) in the \( y \)-direction must be zero: \( \Sigma F_x = 0 \) (Equation 4.9a) and \( \Sigma F_y = 0 \) (Equation 4.9b). These equations will allow us to determine the angle \( \phi \) and the tension \( T \).

SOLUTION
Since the wire is in equilibrium, the sum of the forces in the \( x \) direction is zero:

\[
-2T \sin \phi + F = 0
\]

Substituting in \( F = ILB \sin 90.0^\circ \) for the magnitude of the magnetic force, this equation becomes
\[-2T \sin \phi + ILB \sin 90.0^\circ = 0 \quad (1)\]

The sum of the forces in the y direction is also zero:

\[\frac{+2T \cos \phi - mg}{\Sigma F_y} = 0 \quad (2)\]

Since these two equations contain two unknowns, \( \phi \) and \( T \), we can solve for each of them.

a. To obtain the angle \( \phi \), we solve Equation (2) for the tension \( T = \frac{mg}{2 \cos \phi} \) and substitute the result into Equation (1). This gives

\[-2 \left( \frac{mg}{2 \cos \phi} \right) \sin \phi + ILB \sin 90.0^\circ = 0 \quad \text{or} \quad \frac{\sin \phi}{\cos \phi} = \frac{ILB}{mg} \tan \phi\]

Thus,

\[\phi = \tan^{-1} \left( \frac{ILB}{mg} \right) = \tan^{-1} \left[ \frac{(42 \text{ A})(0.20 \text{ m})(0.070 \text{ T})}{(0.080 \text{ kg})(9.80 \text{ m/s}^2)} \right] = 37^\circ\]

b. The tension in each wire can be found directly from Equation (2):

\[T = \frac{mg}{2 \cos \phi} = \frac{(0.080 \text{ kg})(9.80 \text{ m/s}^2)}{2 \cos 37^\circ} = 0.49 \text{ N}\]

41. **REASONING** The following drawing shows a side view of the conducting rails and the aluminum rod. Three forces act on the rod: (1) its weight \( mg \), (2) the magnetic force \( F \), and the normal force \( F_N \). An application of the Right-Hand Rule No. 1 shows that the magnetic force is directed to the left, as shown in the drawing. Since the rod slides down the rails at a constant velocity, its acceleration is zero. If we choose the x-axis to be along the rails, Newton’s second law states that the net force along the x-direction is zero: \( \Sigma F_x = ma_x = 0 \). Using the components of \( F \) and \( mg \) that are along the x-axis, Newton’s second law becomes

\[\frac{-F \cos 30.0^\circ + mg \sin 30.0^\circ}{\Sigma F_x} = 0\]
The magnetic force is given by Equation 21.3 as \( F = ILB \sin \theta \), where \( \theta = 90.0^\circ \) is the angle between the magnetic field and the current. We can use these two relations to find the current in the rod.

\[ I = \frac{mg \sin 30.0^\circ}{B \sin 90.0^\circ \cos 30.0^\circ} = \frac{0.20 \text{ kg} \times 8.80 \text{ m/s}^2 \times \sin 30.0^\circ}{16 \text{ m/s} \times 0.050 \text{ T} \times \sin 90.0^\circ \cos 30.0^\circ} = 14 \text{ A} \]

42. **REASONING** According to Equation 21.4, the maximum torque is \( \tau_{\text{max}} = NIAB \), where \( N \) is the number of turns in the coil, \( I \) is the current, \( A = \pi r^2 \) is the area of the circular coil, and \( B \) is the magnitude of the magnetic field. We can apply the maximum-torque expression to each coil, noting that \( \tau_{\text{max}}, N, \) and \( I \) are the same for each.

**SOLUTION** Applying Equation 21.4 to each coil, we have

\[ \tau_{\text{max}} = NI\pi r_1^2 B_1 \quad \text{and} \quad \tau_{\text{max}} = NI\pi r_2^2 B_2 \]

Dividing the expression for coil 2 by the expression for coil 1 gives

\[ \frac{\tau_{\text{max}}}{\tau_{\text{max}}} = \frac{NI\pi r_2^2 B_2}{NI\pi r_1^2 B_1} \quad \text{or} \quad 1 = \frac{r_2^2 B_2}{r_1^2 B_1} \]

Solving for \( r_2 \), we obtain

\[ r_2 = r_1 \sqrt{\frac{B_1}{B_2}} = 30 \text{ cm} \times \sqrt{\frac{0.18 \text{ T}}{0.42 \text{ T}}} = 3.3 \text{ cm} \]
43. **REASONING** The magnitude \( \tau \) of the torque that acts on a current-carrying coil placed in a magnetic field is specified by \( \tau = N I A B \sin \phi \) (Equation 21.4), where \( N \) is the number of loops in the coil, \( I \) is the current, \( A \) is the area of one loop, \( B \) is the magnitude of the magnetic field, and \( \phi \) is the angle between the normal to the coil and the magnetic field. All the variables in this relation are known except for the current, which can, therefore, be obtained.

**SOLUTION** Solving the equation \( \tau = N I A B \sin \phi \) for the current \( I \) and noting that \( \phi = 90.0^\circ \) since \( \tau \) is specified to be the maximum torque, we have

\[
I = \frac{\tau}{N A B \sin \phi} = \frac{5.8 \text{ N} \cdot \text{m}}{(1200)(1.1 \times 10^{-2} \text{ m}^2)(0.20 \text{ T})\sin 90.0^\circ} = 2.2 \text{ A}
\]

44. **REASONING** The magnetic moment of a current-carrying coil is discussed in Section 21.6, where it is given as

\[
\text{Magnetic moment} = N I A \tag{1}
\]

In Equation (1), \( N \) is the number of turns in the coil, \( I \) is the current it carries, and \( A \) is its area. Both coils in this problem are circular, so their areas are calculated from their radii via \( A = \pi r^2 \).

**SOLUTION** Because the magnetic moments of the two coils are equal, Equation (1) yields

\[
N_2 I_2 A_2 = N_1 I_1 A_1 \tag{2}
\]

Substituting \( A = \pi r^2 \) into Equation (2) and solving for \( r_2 \), we obtain

\[
N_2 I_2 (\pi r_2^2) = N_1 I_1 (\pi r_1^2) \quad \text{or} \quad r_2 = r_1 \sqrt{\frac{N_1 I_1}{N_2 I_2}} \tag{3}
\]

Therefore, the radius of the second coil is

\[
r_2 = r_1 \sqrt{\frac{N_1 I_1}{N_2 I_2}} = (0.088 \text{ m}) \sqrt{\frac{(140)(4.2 \text{ A})}{(170)(9.5 \text{ A})}} = 0.053 \text{ m}
\]

45. **REASONING** According to Equation 21.4, the torque \( \tau \) that the circular coil experiences is \( \tau = N I A B \sin \phi \), where \( N \) is the number of turns, \( I \) is the current, \( A \) is the area of the circle, \( B \) is the magnetic field strength, and \( \phi \) is the angle between the normal to the coil and the magnetic field. To use this expression, we need the area of the circle, which is \( \pi r^2 \), where \( r \) is the radius. We do not know the radius, but we know the length \( L \) of the wire, which must equal the circumference of the single turn. Thus, \( L = 2\pi r \), which can be solved for the radius.
SOLUTION Using Equation 21.4 and the fact that the area $A$ of a circle is $A = \pi r^2$, we have that

$$
\tau = NIAB \sin \phi = NI \left( \pi r^2 \right) B \sin \phi
$$

(1)

Since the length of the wire is the circumference of the circle, or $L = 2\pi r$, it follows that the radius of the circle is $r = \frac{L}{2\pi}$. Substituting this result into Equation (1) gives

$$
\tau = NI \left( \pi \left( \frac{L}{2\pi} \right)^2 \right) B \sin \phi = \frac{NI^2 B}{4\pi} \sin \phi
$$

The maximum torque $\tau_{\text{max}}$ occurs when $\phi = 90.0^\circ$, so that

$$
\tau_{\text{max}} = \frac{NI^2 B}{4\pi} \sin 90.0^\circ = \frac{(1)(4.30 \text{ A})(7.00 \times 10^{-2} \text{ m})^2 (2.50 \text{ T})}{4\pi} \sin 90.0^\circ = 4.19 \times 10^{-3} \text{ N} \cdot \text{m}
$$

46. REASONING According to the discussion in Section 21.6, the magnetic moment of the current-carrying triangle is $NIA$, where $N = 1$ is the number of loops in the coil, $I$ is the current in the coil, and $A$ is the area of the triangle. The magnitude $\tau$ of the net torque exerted on the triangle by the magnetic field is $\tau = NI \left( B \sin \phi \right)$ (Equation 21.4), where $B$ is the magnitude of the magnetic field and $\phi = 90.0^\circ$ is the angle between the magnetic field and the normal to the plane of the triangle.

SOLUTION

a. Using the fact that the area of a triangle is one-half the base times the height of the triangle, we find that the magnetic moment is

$$
\text{Magnetic moment} = NIA = (1) \left( 4.70 \text{ A} \right) \frac{1}{2} \left( 2.00 \text{ m} \right) \left[ \left( 2.00 \text{ m} \right) \tan 55.0^\circ \right] = 13.4 \text{ A} \cdot \text{m}^2
$$

b. The magnitude of the net torque exerted on the triangle is

$$
\tau = \frac{NI}{\text{Magnetic moment}} \left( B \sin \phi \right) = \left( 13.4 \text{ A} \cdot \text{m}^2 \right) \left( 1.80 \text{ T} \right) \sin 90.0^\circ = 24.1 \text{ N} \cdot \text{m}
$$

(21.4)

47. REASONING The magnitude $\tau$ of the torque that acts on a current-carrying coil placed in a magnetic field is given by $\tau = NIAB \sin \phi$ (Equation 21.4), where $N$ is the number of loops in the coil ($N = 1$ in this problem), $I$ is the current, $A$ is the area of one loop, $B$ is the magnitude of the magnetic field (the same for each coil), and $\phi$ is the angle (the same for each coil) between the normal to the coil and the magnetic field. Since we are given that the torque for the square coil is the same as that for the circular coil, we can write
\[
\frac{(1) I_{\text{square}} A_{\text{square}} B \sin \phi}{\tau_{\text{square}}} = \frac{(1) I_{\text{circle}} A_{\text{circle}} B \sin \phi}{\tau_{\text{circle}}}
\]

This relation can be used directly to find the ratio of the currents.

**SOLUTION** Solving the equation above for the ratio of the currents yields

\[
\frac{I_{\text{square}}}{I_{\text{circle}}} = \frac{A_{\text{circle}}}{A_{\text{square}}}
\]

If the length of each wire is \(L\), the length of each side of the square is \(\frac{1}{4} L\), and the area of the square coil is \(A_{\text{square}} = \left(\frac{1}{4} L\right)\left(\frac{1}{4} L\right) = \frac{1}{16} L^2\). The area of the circular coil is \(A_{\text{circle}} = \pi r^2\), where \(r\) is the radius of the coil. Since the circumference \((2\pi r)\) of the circular coil is equal to the length \(L\) of the wire, we have \(2\pi r = L\), or \(r = L/(2\pi)\). Substituting this value for \(r\) into the expression for the area of the circular coil gives \(A_{\text{circle}} = \pi \left[ L/(2\pi) \right]^2\). Thus, the ratio of the currents is

\[
\frac{I_{\text{square}}}{I_{\text{circle}}} = \frac{A_{\text{circle}}}{A_{\text{square}}} = \frac{\pi \left(\frac{L}{2\pi}\right)^2}{\frac{1}{16} L^2} = \frac{4}{\pi} = 1.27
\]

48. **REASONING** When the wire is used to make a single-turn square coil, each side of the square has a length of \(\frac{1}{4} L\). When the wire is used to make a two-turn coil, each side of the square has a length of \(\frac{1}{8} L\). The drawing shows these two options and indicates that the total effective area of \(NA\) is greater for the single-turn option. Hence, more torque is obtained by using the single-turn option.

**SOLUTION** According to Equation 21.4 the maximum torque experienced by the coil is \(\tau_{\text{max}} = (NIAB) \sin 90^\circ = NIAB\), where \(N\) is the number of turns, \(I\) is the current, \(A\) is the area of each turn, and \(B\) is the magnitude of the magnetic field. Applying this expression to each option gives

**Single-turn** \(\tau_{\text{max}} = NIAB = NI \left(\frac{1}{4} L\right)^2 B\)

\[
= (1)(1.7 A) \left[\frac{1}{4} (1.00 \text{ m})\right]^2 (0.34 \text{ T}) = 0.036 \text{ N} \cdot \text{m}
\]

**Two-turn** \(\tau_{\text{max}} = NIAB = NI \left(\frac{1}{8} L\right)^2 B\)

\[
= (2)(1.7 A) \left[\frac{1}{8} (1.00 \text{ m})\right]^2 (0.34 \text{ T}) = 0.018 \text{ N} \cdot \text{m}
\]
49. **REASONING** The torque on the loop is given by Equation 21.4, \( \tau = NIAB \sin \phi \).
   From the drawing in the text, we see that the angle \( \phi \) between the normal to the plane of the loop and the magnetic field is \( 90^\circ - 35^\circ = 55^\circ \). The area of the loop is \( 0.70 \text{ m} \times 0.50 \text{ m} = 0.35 \text{ m}^2 \).

   **SOLUTION**
   a. The magnitude of the net torque exerted on the loop is
   \[
   \tau = NIAB \sin \phi = (75)(4.4 \text{ A})(0.35 \text{ m}^2 )(1.8 \text{ T}) \sin 55^\circ = 170 \text{ N} \cdot \text{m}
   \]
   b. As discussed in the text, when a current-carrying loop is placed in a magnetic field, the loop tends to rotate such that its normal becomes aligned with the magnetic field. The normal to the loop makes an angle of \( 55^\circ \) with respect to the magnetic field. Since this angle decreases as the loop rotates, the \( 35^\circ \) angle increases.

50. **REASONING AND SOLUTION** According to Equation 21.4, the maximum torque for a single turn is \( \tau_{\text{max}} = IAB \). When the length \( L \) of the wire is used to make the square, each side of the square has a length \( L/4 \). The area of the square is \( A_{\text{square}} = (L/4)^2 \). For the rectangle, since two sides have a length \( d \), while the other two sides have a length \( 2d \), it follows that \( L = 6d \), or \( d = L/6 \). The area is \( A_{\text{rectangle}} = 2d^2 = 2(L/6)^2 \). Using Equation 21.4 for the square and the rectangle, we obtain
   \[
   \frac{\tau_{\text{square}}}{\tau_{\text{rectangle}}} = \frac{IA_{\text{square}}B}{IA_{\text{rectangle}}B} = \frac{A_{\text{square}}}{A_{\text{rectangle}}} = \frac{5}{3} \approx 1.13
   \]

51. **REASONING** The coil in the drawing is oriented such that the normal to the surface of the coil is perpendicular to the magnetic field (\( \phi = 90^\circ \)). The magnetic torque is a maximum, and Equation 21.4 gives its magnitude as \( \tau = NIAB \sin \phi \). In this expression \( N \) is the number of loops in the coil, \( I \) is the current, \( A \) is the area of one loop, and \( B \) is the magnitude of the magnetic field. The torque from the brake balances this magnetic torque. The brake torque is \( \tau_{\text{brake}} = F_{\text{brake}}r \), where \( F_{\text{brake}} \) is the brake force, and \( r \) is the radius of the shaft and also the lever arm. The maximum value for the brake force available from static friction is \( F_{\text{brake}} = \mu_s F_N \) (Equation 4.7), where \( F_N \) is the normal force pressing the brake shoe against the shaft. The maximum brake torque, then, is \( \tau_{\text{brake}} = \mu_s F_N r \). By setting \( \tau_{\text{brake}} = \tau_{\text{max}} \), we will be able to determine the magnitude of the normal force.

   **SOLUTION** Setting the torque produced by the brake equal to the maximum torque produced by the coil gives
\[ \tau_{\text{brake}} = \tau \quad \text{or} \quad \mu_s F_N r = N I A B \sin \phi \]

\[ F_N = \frac{N I A B \sin \phi}{\mu_s r} = \frac{(410)(0.26 \text{ A})(3.1 \times 10^{-3} \text{ m}^2)(0.23 \text{ T})\sin 90^\circ}{(0.76)(0.012 \text{ m})} = 8.3 \text{ N} \]

52. **REASONING** The magnetic moment of the orbiting electron can be found from the expression \( \text{Magnetic moment} = N I A \). For this situation, \( N = 1 \). Thus, we need to find the current and the area for the orbiting charge.

**SOLUTION** The current for the orbiting charge is, by definition (see Equation 20.1),

\[ I = \frac{\Delta q}{\Delta t} \]

where \( \Delta q \) is the amount of charge that passes a given point during a time interval \( \Delta t \). Since the charge (\( \Delta q = e \)) passes by a given point once per revolution, we can find the current by dividing the total orbiting charge by the period \( T \) of revolution.

\[ I = \frac{\Delta q}{T} = \frac{\Delta q}{2\pi r/v} = \frac{(1.6\times10^{-19} \text{ C})(2.2\times10^6 \text{ m/s})}{2\pi(5.3\times10^{-11} \text{ m})} = 1.06 \times 10^{-3} \text{ A} \]

The area of the orbiting charge is

\[ A = \pi r^2 = \pi (5.3 \times 10^{-11} \text{ m})^2 = 8.82 \times 10^{-21} \text{ m}^2 \]

Therefore, the magnetic moment is

\[ \text{Magnetic moment} = N I A = (1)(1.06 \times 10^{-3} \text{ A})(8.82 \times 10^{-21} \text{ m}^2) = 9.3 \times 10^{-24} \text{ A} \cdot \text{m}^2 \]

53. **SSM REASONING AND SOLUTION**

a. In Figure 21.27a the magnetic field that exists at the location of each wire points upward. Since the current in each wire is the same, the fields at the locations of the wires also have the same magnitudes. Therefore, a single external field that points downward will cancel the mutual repulsion of the wires, if this external field has a magnitude that equals that of the field produced by either wire.

b. Equation 21.5 gives the magnitude of the field produced by a long straight wire. The external field must have this magnitude:

\[ B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(25 \text{ A})}{2\pi(0.016 \text{ m})} = 3.1 \times 10^{-4} \text{ T} \]
54. **REASONING** The magnitude $B$ of the magnetic field in the interior of a long solenoid is $B = \mu_0 n I$ (Equation 21.7), where $\mu_0 = 4\pi \times 10^{-7}$ T · m/A is the permeability of free space, $n$ is the number of turns per unit length of the solenoid, and $I$ is the current.

**SOLUTION** Using Equation 21.7, we find that

$$B = \mu_0 n I = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \left( \frac{1400 \text{ turns}}{0.65 \text{ m}} \right)(4.7 \text{ A}) = 1.3 \times 10^{-2} \text{ T}$$

55. **SSM REASONING** The magnitude $B$ of the magnetic field in the interior of a solenoid that has a length much greater than its diameter is given by $B = \mu_0 n I$ (Equation 21.7), where $\mu_0 = 4\pi \times 10^{-7}$ T · m/A is the permeability of free space, $n$ is the number of turns per meter of the solenoid's length, and $I$ is the current in the wire of the solenoid. Since $B$ and $I$ are given, we can solve Equation 21.7 for $n$.

**SOLUTION** Solving Equation 21.7 for $n$, we find that the number of turns per meter of length is

$$n = \frac{B}{\mu_0 I} = \frac{7.0 \text{ T}}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.0 \times 10^2 \text{ A})} = 2.8 \times 10^4 \text{ turns/m}$$

56. **REASONING** The torque $\tau$ exerted on a coil with a current $I_{\text{coil}}$ is given by $\tau = N I_{\text{coil}} A B \sin \phi$ (Equation 21.4), where $N$ is the number of turns in the coil, $A$ is its area, $B$ is the magnitude of the external magnetic field causing the torque, and $\phi$ is the angle between the normal to the surface of the coil and the direction of the external magnetic field. The external magnetic field $B$, in this case, is the magnetic field of the solenoid. We will use $B = \mu_0 n I$ (Equation 21.7) to determine the magnetic field, where $\mu_0 = 4\pi \times 10^{-7}$ T · m/A is the permeability of free space, $n$ is the number of turns per meter of length of the solenoid, and $I$ is the current in the solenoid. The magnetic field in the interior of a solenoid is parallel to the solenoid's axis. Because the normal to the surface of the coil is perpendicular to the axis of the solenoid, the angle $\phi$ is equal to 90.0°.

**SOLUTION** Substituting the expression $B = \mu_0 n I$ (Equation 21.7) for the magnetic field of the solenoid into $\tau = N I_{\text{coil}} A B \sin \phi$ (Equation 21.4), we obtain

$$\tau = N I_{\text{coil}} A B \sin \phi = N I_{\text{coil}} A (\mu_0 n I) \sin \phi = \mu_0 n N A I_{\text{coil}} I \sin \phi$$

Therefore, the torque exerted on the coil is

$$\tau = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1400 \text{ m}^{-1})(50)(1.2 \times 10^{-3} \text{ m}^2)(0.50 \text{ A})(3.5 \text{ A}) \sin 90.0°$$

$$= 1.8 \times 10^{-4} \text{ N} \cdot \text{m}$$
57. **REASONING** The magnitude of the magnetic field at the center of a circular loop of current is given by Equation 21.6 as \( B = N\mu_0 I/(2R) \), where \( N \) is the number of turns, \( \mu_0 \) is the permeability of free space, \( I \) is the current, and \( R \) is the radius of the loop. The field is perpendicular to the plane of the loop. Magnetic fields are vectors, and here we have two fields, each perpendicular to the plane of the loop producing it. Therefore, the two field vectors are perpendicular, and we must add them as vectors to get the net field. Since they are perpendicular, we can use the Pythagorean theorem to calculate the magnitude of the net field.

**SOLUTION** Using Equation 21.6 and the Pythagorean theorem, we find that the magnitude of the net magnetic field at the common center of the two loops is

\[
B_{\text{net}} = \sqrt{\left( \frac{2\mu_0 I}{2R} \right)^2 + \left( \frac{2\mu_0 I}{2R} \right)^2} = \sqrt{2} \left( \frac{\mu_0 I}{2R} \right) = \frac{\mu_0 I}{\sqrt{2}R}
\]

\[
= \frac{\sqrt{2} \times 10^{-7}}{2 \times 0.040 \text{ m}} \times 10^{-7} \text{ T} \cdot \text{m/A} = 7 \times 10^{-5} \text{ T} = 3.8 \times 10^{-5} \text{ T}
\]

58. **REASONING** The two rods attract each other because they each carry a current \( I \) in the same direction. The bottom rod floats because it is in equilibrium. The two forces that act on the bottom rod are the downward force of gravity \( mg \) and the upward magnetic force of attraction to the upper rod. If the two rods are a distance \( s \) apart, the magnetic field generated by the top rod at the location of the bottom rod is (see Equation 21.5) \( B = \mu_0 I/(2\pi s) \). According to Equation 21.3, the magnetic force exerted on the bottom rod is \( F = \mu_0 I^2 L \sin \theta / (2\pi s) \), where \( \theta \) is the angle between the magnetic field at the location of the bottom rod and the direction of the current in the bottom rod. Since the rods are parallel, the magnetic field is perpendicular to the direction of the current (RHR-2), and \( \theta = 90.0^\circ \), so that \( \sin \theta = 1.0 \).

**SOLUTION** Taking upward as the positive direction, the net force on the bottom rod is

\[
\frac{\mu_0 I^2 L \sin \theta}{2\pi s} - mg = 0
\]

Solving for \( I \), we find

\[
I = \sqrt{\frac{2\pi mgs}{\mu_0 L}} = \sqrt{\frac{2\pi (0.073 \text{ kg})(9.80 \text{ m/s}^2)(8.2 \times 10^{-3} \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(0.85 \text{ m})}} = 190 \text{ A}
\]
59. **REASONING AND SOLUTION** Let the current in the left-hand wire be labeled \( I_1 \) and that in the right-hand wire \( I_2 \).

a. At point \( A \): \( B_1 \) is up and \( B_2 \) is down, so we subtract them to get the net field. We have

\[
B_1 = \mu_0 I_1 / (2\pi d_1) = \mu_0 (8.0 \text{ A}) / [2\pi (0.030 \text{ m})] \\
B_2 = \mu_0 I_2 / (2\pi d_2) = \mu_0 (8.0 \text{ A}) / [2\pi (0.150 \text{ m})]
\]

So the net field at point \( A \) is

\[
B_A = B_1 - B_2 = 4.3 \times 10^{-5} \text{ T}
\]

b. At point \( B \): \( B_1 \) and \( B_2 \) are both down so we add the two. We have

\[
B_1 = \mu_0 (8.0 \text{ A}) / [2\pi (0.060 \text{ m})] \\
B_2 = \mu_0 (8.0 \text{ A}) / [2\pi (0.060 \text{ m})]
\]

So the net field at point \( B \) is

\[
B_B = B_1 + B_2 = 5.3 \times 10^{-5} \text{ T}
\]

---

60. **REASONING** The net magnetic field is the sum of the uniform field and the field produced by the current in the wire. In order for the net field to be zero, the two field contributions must have the same magnitude and opposite directions. The current \( I \) in the wire creates a field that has a magnitude \( B \) that is given by \( B = \frac{\mu_0 I}{2\pi r} \) (Equation 21.5), where \( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \) is the permeability of free space and \( r \) is the perpendicular distance from the wire. We can solve this equation for \( r \), in order to locate the point where the field produced by the current has a magnitude of \( 7.00 \times 10^{-3} \text{ T} \). Right-hand rule no. 2 indicates that the field of the current has opposite directions on opposite sides of the wire. Therefore, since the wire is perpendicular to the uniform field, we can be confident that this value for \( r \) will locate a place on one side or the other of the wire where the net field is zero.

**SOLUTION** Solving Equation 21.5 for \( r \), we find that

\[
r = \frac{\mu_0 I}{2\pi B} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(305 \text{ A})}{2\pi (7.00 \times 10^{-3} \text{ T})} = 8.71 \times 10^{-3} \text{ m}
\]

---

61. **REASONING** The magnitude \( B_i \) of the magnetic field at the center of the inner coil is given by Equation 21.6 as \( B_i = \mu_0 I_i N_i / (2R_i) \), where \( I_i, N_i, \) and \( R_i \) are, respectively, the current, the number of turns, and the radius of the inner coil. The magnitude \( B_o \) of the magnetic field at the center of the outer coil is \( B_o = \mu_0 I_o N_o / (2R_o) \). In order that the net magnetic field at the common center of the two coils be zero, the individual magnetic fields must have the same
magnitude, but opposite directions. Equating the magnitudes of the magnetic fields produced by the inner and outer coils will allow us to find the current in the outer coil.

**SOLUTION** Setting $B_i = B_o$ gives

$$\mu_0 I_i N_i \frac{R_o}{2R_i} = \mu_0 I_o N_o \frac{R_i}{2R_o}$$

Solving this expression for the current in the outer coil, we have

$$I_o = I_i \left( \frac{N_i}{N_o} \right) \left( \frac{R_o}{R_i} \right) = (7.2 \, \text{A}) \left( \frac{140 \, \text{turns}}{180 \, \text{turns}} \right) \left( \frac{0.023 \, \text{m}}{0.015 \, \text{m}} \right) = 8.6 \, \text{A}$$

In order that the two magnetic fields have opposite directions, the current in the outer coil must have an opposite direction to the current in the inner coil.

62. **REASONING**

a. The compass needle lines up with the net horizontal magnetic field $\mathbf{B}$ that is the vector sum of the magnetic fields $\mathbf{B}_i$ (the field at the location of the compass due to the current $I$ in the wire) and $\mathbf{B}_E$ (the horizontal component of the earth’s magnetic field): $\mathbf{B} = \mathbf{B}_i + \mathbf{B}_E$. The field lines of the magnetic field created by the current $I$ are circles centered on the wire. Because the wire is perpendicular to the earth’s surface, and the compass is directly north of the wire, the magnetic field $\mathbf{B}_i$ due to the wire must point either due east or due west at the location of the compass, depending on the direction of the current. The magnetic field $\mathbf{B}_i$ causes the compass needle to deflect east of north, so we conclude that $\mathbf{B}_i$ points due east (see the drawing, which shows the situation as viewed from above). We will use Right-Hand Rule No. 2 to determine the direction of the current $I$ from the direction of $\mathbf{B}_i$.

b. The horizontal component $\mathbf{B}_E$ of the earth’s magnetic field points north, so it is perpendicular to the magnetic field $\mathbf{B}_i$ created by the current in the wire, which points east. Therefore, the vectors $\mathbf{B}$, $\mathbf{B}_E$, and $\mathbf{B}_i$ form a right triangle with $\mathbf{B}$ serving as the hypotenuse (see the drawing). The angle $\theta$ between the vectors $\mathbf{B}_E$ and $\mathbf{B}$, then, is given by $\tan \theta = \frac{B_i}{B_E}$ (Equation 1.3). We will use $B = \mu_0 \frac{I}{2\pi r}$ (Equation 21.5) to determine $B_i$, where $r = 0.280 \, \text{m}$ is
the radial distance between the wire and the center of the compass needle and 
\( \mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \). Then we will use Equation 1.3 to find the magnitude \( B_E \) of the horizontal component of earth's magnetic field.

**SOLUTION**
a. Because the magnetic field \( B_1 \) due to the current in the wire points east at the location of the compass, the magnetic field lines must circulate clockwise around the wire, as viewed from above. Right-Hand Rule No. 2, then, indicates that the current \( I \) flows into the page. Since the drawing shows a top view of the situation, the current flows toward the earth's surface.

b. Solving \( \tan \theta = \frac{B_1}{B_E} \) (Equation 1.3) for \( B_E \), we obtain

\[
B_E = \frac{B_1}{\tan \theta}
\]

Substituting \( B_1 = \frac{\mu_0 I}{2\pi r} \) (Equation 21.5) into Equation (1), we find that

\[
B_E = \frac{B_1}{\tan \theta} = \frac{\left( \frac{\mu_0 I}{2\pi r} \right)}{\tan \theta} = \frac{\mu_0 I}{2\pi r \tan \theta} = \frac{\left( 4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \right)(25.0 \text{ A})}{2\pi \left( 0.280 \text{ m} \right) \tan 23.0^\circ} = 4.21 \times 10^{-5} \text{T}
\]

63. **REASONING** The drawing shows an end-on view of the two wires, with the currents directed out of the plane of the paper toward you. \( B_1 \) and \( B_2 \) are the individual fields produced by each wire. Applying RHR-2 (thumb points in the direction of the current, fingers curl into the shape of a half-circle and the finger tips point in the direction of the field), we obtain the field directions shown in the drawing for each of the three regions mentioned in the problem statement. Note that it is only in the region between the wires that \( B_1 \) and \( B_2 \) have opposite directions. Hence, the spot where the net magnetic field (the vector sum of the individual fields) is zero must lie between the wires. At this spot the magnitudes of the fields \( B_1 \) and \( B_2 \) from the wires are equal and each is given by \( B = \frac{\mu_0 I}{2\pi r} \) (Equation 21.5), where \( \mu_0 \) is the permeability of free space, \( I \) is the current in the wire, and \( r \) is the distance from the wire.

**SOLUTION** The spot we seek is located at a distance \( d \) from wire 1 (see the drawing). Note that the wires are separated by a distance of one meter. Applying Equation 21.5 to each wire, we have that
\[ B_1 = B_2 \quad \text{or} \quad \frac{\mu_0 I_1}{2\pi d} = \frac{\mu_0 I_2}{2\pi (1.00 \text{ m} - d)} \]

Since \( I_1 = 4I_2 \), this result becomes

\[ \frac{\mu_0 (4I_2)}{2\pi d} = \frac{\mu_0 I_2}{2\pi (1.00 \text{ m} - d)} \quad \text{or} \quad \frac{4}{d} = \frac{1}{1.00 \text{ m} - d} \quad \text{or} \quad 4(1.00 \text{ m} - d) = d \]

Solving for \( d \), we find that \( d = 0.800 \text{ m} \)

64. **Reasoning** Using Right-Hand Rule No. 2, we can see that at point \( A \) the magnetic field due to the horizontal current points perpendicularly out of the plane of the paper. Similarly, the magnetic field due to the vertical current points perpendicularly into the plane of the paper. Point \( A \) is closer to the horizontal wire, so that the effect of the horizontal current dominates and the net field is directed out of the plane of the paper.

Using Right-Hand Rule No. 2, we can see that at point \( B \) the magnetic field due to the horizontal current points perpendicularly into the plane of the paper. Similarly, the magnetic field due to the vertical current points perpendicularly out of the plane of the paper. Point \( B \) is closer to the horizontal wire, so that the effect of the horizontal current dominates and the net field is directed into the plane of the paper.

Points \( A \) and \( B \) are the same distance of 0.20 m from the horizontal wire. They are also the same distance of 0.40 m from the vertical wire. Therefore, the magnitude of the field contribution from the horizontal current is the same at both points, although the directions of the field contributions are opposite. Likewise, the magnitude of the field contribution from the vertical current is the same at both points, although the directions of the field contributions are opposite. At either point the magnitude of the net field is the magnitude of the difference between the two contributions, and this is the same at points \( A \) and \( B \).

**Solution** According to Equation 21.5, the magnitude of the magnetic field from the current in a long straight wire is \( B = \frac{\mu_0 I}{2\pi r} \), where \( \mu_0 \) is the permeability of free space, \( I \) is the current, and \( r \) is the distance from the wire. Applying this equation to each wire at each point, we see that the magnitude of the net field \( B_{\text{net}} \) is

**Point A**

\[
B_{\text{net}} = \frac{\mu_0 I}{2\pi r_{A, H}} - \frac{\mu_0 I}{2\pi r_{A, V}} = \frac{\mu_0 I}{2\pi} \left( \frac{1}{r_{A, H}} - \frac{1}{r_{A, V}} \right)
\]

\[
= \left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) (5.6 \text{ A}) \left( \frac{1}{0.20 \text{ m}} - \frac{1}{0.40 \text{ m}} \right) = 2.8 \times 10^{-6} \text{ T}
\]
65. **SSM REASONING** According to Equation 21.6 the magnetic field at the center of a circular, current-carrying loop of $N$ turns and radius $r$ is $B = N\mu_0 I/(2r)$. The number of turns $N$ in the coil can be found by dividing the total length $L$ of the wire by the circumference after it has been wound into a circle. The current in the wire can be found by using Ohm's law, $I = V/R$.

**SOLUTION** The number of turns in the wire is

$$N = \frac{L}{2\pi r}$$

The current in the wire is

$$I = \frac{V}{R} = \frac{12.0 \text{ V}}{(5.90 \times 10^{-3} \Omega/\text{m})L} = \frac{2.03 \times 10^3}{L} \text{ A}$$

Therefore, the magnetic field at the center of the coil is

$$B = N\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0 LI}{4\pi r^2}$$

$$= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(5.6 \text{ A})}{2\pi} \left(\frac{1}{0.20 \text{ m}} - \frac{1}{0.40 \text{ m}}\right) = 2.8 \times 10^{-6} \text{ T}$$

66. **REASONING** In the drawing at the right, we have labeled the current on the left as $I_1$ and its distance from point $P$ as $r_1$. Similarly, we have labeled the current on the right as $I_2$ and its distance from point $P$ as $r_2$. In fact, however, as the drawing in the text indicates, $I_1 = I_2$ and $r_1 = r_2$ (the dashed triangle is an equilateral triangle). It is important to note that within the triangle the angle between $r_1$ and the $-y$ axis is $30^\circ$, as is the angle between $r_2$ and the $-y$ axis, because the $-y$ axis bisects the $60^\circ$ apex angle of the triangle. Moreover, it follows from right-hand rule no. 2 that the magnetic field $B_1$ (from current $I_1$) is perpendicular to $r_1$ and also that the magnetic field $B_2$ (from current $I_2$) is perpendicular to $r_2$. This, in turn, means that the fields $B_1$ and $B_2$ make
60° angles with the −y axis, as indicated in the drawing. We will use this geometry in dealing with the components of the two magnetic fields. It is necessary to use components to find the net field at point P, since the fields are vectors.

**SOLUTION** The current I in each wire creates a field that has a magnitude B that is given by \( B = \frac{\mu_0 I}{2\pi r} \) (Equation 21.5), where \( \mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \) is the permeability of free space and r is the perpendicular distance from the wire. Using Equation 21.5 for the magnetic fields \( B_1 \) and \( B_2 \), we list the components of the fields at point P and the net components as follows:

<table>
<thead>
<tr>
<th>Field</th>
<th>( x ) component</th>
<th>( y ) component</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_1 )</td>
<td>( B_{1x} = B_1 \sin 60.0° = \left( \frac{\mu_0 I}{2\pi r} \right) \sin 60.0° )</td>
<td>( B_{1y} = -B_1 \cos 60.0° = -\left( \frac{\mu_0 I}{2\pi r} \right) \cos 60.0° )</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>( B_{2x} = -B_2 \sin 60.0° = -\left( \frac{\mu_0 I}{2\pi r} \right) \sin 60.0° )</td>
<td>( B_{2y} = -B_2 \cos 60.0° = -\left( \frac{\mu_0 I}{2\pi r} \right) \cos 60.0° )</td>
</tr>
<tr>
<td>Net</td>
<td>0</td>
<td>(-2\left( \frac{\mu_0 I}{2\pi r} \right) \cos 60.0° )</td>
</tr>
</tbody>
</table>

The net field at point P has a zero \( x \) component, so its magnitude is just the magnitude of the \( y \) component:

\[
\text{Magnitude of net field at point } P = 2 \left( \frac{\mu_0 I}{2\pi r} \right) \cos 60.0° = 2 \left( \frac{4\pi \times 10^{-7} \text{T} \cdot \text{m/A}}{2\pi(0.150 \text{ m})} \right)(85.0 \text{ A}) \cos 60.0° = 1.13 \times 10^{-4} \text{T}
\]

Since the net field has only a \( y \) component that is negative, the net field points downward and is perpendicular to the dashed line that is the base of the triangle in our drawing.

67. **REASONING AND SOLUTION** The forces acting on each wire are the magnetic force \( F \), the gravitational force \( mg \), and the tension \( T \) in the strings. Each string makes an angle of 7.5° with respect to the vertical. From the drawing below at the right we can relate the magnetic force to the gravitational force. Since the wire is in equilibrium, Newton’s second law requires that \( \Sigma F_x = 0 \) and \( \Sigma F_y = 0 \). These equations become

\[
\sum F_x = -T \sin 7.5° - F = 0 \quad \text{and} \quad \sum F_y = T \cos 7.5° - mg = 0
\]
Solving the first equation for \( T \), and then substituting the result into the second equation gives (after some simplification)

\[
\tan 7.5^\circ = \frac{F}{mg}
\]  

(1)

The magnetic force \( F \) exerted on one wire by the other is \( F = \frac{\mu_0 I^2 L}{2\pi d} \), where \( d \) is the distance between the wires \([d/2 = (1.2 \text{ m}) \sin 7.5^\circ, \text{ so that } d = 0.31 \text{ m}]\), \( I \) is the current (which is the same for each wire), and \( L \) is the length of each wire. Substituting this relation for \( F \) into Equation (1) and then solving for the current, gives

\[
I = \sqrt{\frac{\mu_0 L}{\tan 7.5^\circ}} \left( \frac{L}{\frac{1}{2} \frac{0.31 \text{ m}}{0.050 \text{ kg/m} \cdot 80 \text{ m/s}^2 \cdot \tan 7.5^\circ}} \right)
\]

\[
= \sqrt{\frac{0.050 \text{ kg/m}}{0.80 \text{ m/s}^2 \cdot \tan 7.5^\circ}} \left( \frac{2 \cdot 0.31 \text{ m}}{\mu_0} \right) \left( \frac{2 \cdot 0.31 \text{ m}}{\mu_0} \right) \quad 320 \text{ A}
\]

68. **REASONING AND SOLUTION**  Ampère's law in the form of Equation 21.8 indicates that \( \Sigma B \Delta \ell = \mu_0 I \). Since the magnetic field is everywhere perpendicular to the plane of the paper, it is everywhere perpendicular to the circular path and has no component \( B_\parallel \) that is parallel to the circular path. Therefore, Ampère's law reduces to \( \Sigma B_\parallel \Delta \ell = 0 = \mu_0 I \), so that the net current passing through the circular surface is zero.

69. **REASONING**  Since the two wires are next to each other, the net magnetic field is everywhere parallel to \( \Delta \ell \) in Figure 21.38. Moreover, the net magnetic field \( B \) has the same magnitude \( B \) at each point along the circular path, because each point is at the same distance from the wires. Thus, in Ampère's law (Equation 21.8), \( B_\parallel = B \), \( I = I_1 + I_2 \), and we have

\[
\Sigma B_\parallel \Delta \ell = B \left( \Sigma \Delta \ell \right) = \mu_0 \left( I_1 + I_2 \right)
\]

But \( \Sigma \Delta \ell \) is just the circumference \((2\pi r)\) of the circle, so Ampère's law becomes

\[
B(2\pi r) = \mu_0 \left( I_1 + I_2 \right)
\]

This expression can be solved for \( B \).

**SOLUTION**
a. When the currents are in the same direction, we find that

\[
B = \frac{\mu_0 (I_1 + I_2)}{2\pi r} = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(28 \text{ A} + 12 \text{ A})}{2\pi (0.72 \text{ m})} = 1.1 \times 10^{-5} \text{ T}
\]

b. When the currents have opposite directions, a similar calculation shows that

\[
B = \frac{\mu_0 (I_1 - I_2)}{2\pi r} = \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(28 \text{ A} - 12 \text{ A})}{2\pi (0.72 \text{ m})} = 4.4 \times 10^{-6} \text{ T}
\]

70. **REASONING** Both parts of this problem can be solved using Ampère’s law with the circular closed paths suggested in the *Hint* given with the problem statement. The circular closed paths are used because of the symmetry in the way the current is distributed on the copper cylinder.

**SOLUTION**

a. An end-on view of the copper cylinder is a circle, as the drawing at the right shows. The dots around the circle represent the current coming out of the paper toward you. The larger dashed circle of radius \( r \) is the closed path used in Ampère’s law and is centered on the axis of the cylinder. Equation 21.8 gives Ampère’s law as \( \Sigma B_{\parallel} \Delta \ell = \mu_0 I \). Because of the symmetry of the arrangement in the drawing, we have \( B_{\parallel} = B \) for all \( \Delta \ell \) on the circular path, so that Ampère’s law becomes

\[
\Sigma B_{\parallel} \Delta \ell = B(\Sigma \Delta \ell) = \mu_0 I
\]

In this result, \( I \) is the net current through the circular surface bounded by the dashed path. In other words, it is the current \( I \) in the copper tube. Furthermore, \( \Sigma \Delta \ell \) is the circumference of the circle, so we find that

\[
B(\Sigma \Delta \ell) = B(2\pi r) = \mu_0 I \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi r}
\]
b. The setup here is similar to that in part a, except that the smaller dashed circle of radius \( r \) is now the closed path used in Ampère's law (see the drawing at the right). With this change, the derivation then proceeds exactly as in part a. Now, however, there is no current through the circular surface bounded by the dashed path, because all of the current is outside the path. Therefore, \( I = 0 \) A, and

\[
B = \frac{\mu_0 I}{2\pi r} = 0 \text{ T}
\]

71. SSM REASONING AND SOLUTION The drawing at the right shows an end-on view of the solid cylinder. The dots represent the current in the cylinder coming out of the paper toward you. The dashed circle of radius \( r \) is the closed path used in Ampère's law and is centered on the axis of the cylinder. Equation 21.8 gives Ampère's law as \( \Sigma B \Delta \ell = \mu_0 I \). Because of the symmetry of the arrangement in the drawing, we have \( B \parallel = B \) for all \( \Delta \ell \) on the circular path, so that Ampère's law becomes

\[
\Sigma B \Delta \ell = B(\Sigma \Delta \ell) = \mu_0 I
\]

In this result, \( \Sigma \Delta \ell = 2\pi r \), the circumference of the circle. The current \( I \) is the part of the total current that comes through the area \( \pi r^2 \) bounded by the dashed path. We can calculate this current by using the current per unit cross-sectional area of the solid cylinder. This current per unit area is called the current density. The current \( I \) is the current density times the area \( \pi r^2 \):

\[
I = \left( \frac{I_0}{\pi R^2} \right)(\pi r^2) = \frac{I_0 r^2}{R^2}
\]

Thus, Ampère's law becomes

\[
B(\Sigma \Delta \ell) = \mu_0 I \quad \text{or} \quad B(2\pi r) = \mu_0 \left( \frac{I_0 r^2}{R^2} \right) \quad \text{or} \quad B = \frac{\mu_0 I_0 r}{2\pi R^2}
\]
72. **REASONING** The magnitude \(|q_0|\) of the electric charge of the bullet is related to the magnitude \(F\) of the magnetic force exerted on the bullet according to \(B = \frac{F}{|q_0| (v \sin \theta)}\) (Equation 21.1).

Here, \(\theta\) is the angle between the direction of the bullet's velocity \(v\) and the earth's magnetic field \(B\). (See the drawing, which depicts the situation as seen from the side.) We will find the magnitude of the bullet's charge from Equation 21.1, and Right-Hand Rule No. 1 will allow us to determine the algebraic sign of the charge.

**SOLUTION** Solving \(B = \frac{F}{|q_0| (v \sin \theta)}\) (Equation 21.1) for \(|q_0|\), we obtain

\[
|q_0| = \frac{F}{B (v \sin \theta)}
\]  

(1)

The angle \(\theta\) is the angle between the directions of the bullet's velocity \(v\) and the direction of earth's magnetic field \(B\). Therefore, \(\theta\) is the sum of the angles that these vectors make with the horizontal (see the drawing):

\[
\theta = 58^\circ + 11^\circ = 69^\circ
\]

Applying Right-Hand Rule No. 1 to the vectors \(v\) and \(B\) (see the drawing), we see that the force on a positively charged bullet would point into the page, which is west. Because the force on this bullet points to the east, out of the page, the bullet's charge must be negative. When removing the absolute value brackets from Equation (1), therefore, we must insert a minus sign on the right hand side. Making this change, we find that

\[
q_0 = \frac{F}{B (v \sin \theta)} = -\frac{2.8 \times 10^{-10} \text{ N}}{(5.4 \times 10^{-5} \text{ T})(670 \text{ m/s}) \sin 69^\circ} = -8.3 \times 10^{-9} \text{ C}
\]

73. **SSM REASONING** The angle \(\theta\) between the electron's velocity and the magnetic field can be found from Equation 21.1,

\[
\sin \theta = \frac{F}{|q| v B}
\]

According to Newton's second law, the magnitude \(F\) of the force is equal to the product of the electron's mass \(m\) and the magnitude \(a\) of its acceleration, \(F = ma\).
**SOLUTION** The angle $\theta$ is

$$\theta = \sin^{-1} \left( \frac{ma}{|q|vB} \right) = \sin^{-1} \left[ \frac{\left( 9.11 \times 10^{-31} \text{ kg} \right) \left( 3.50 \times 10^{14} \text{ m/s}^2 \right)}{\left( 1.60 \times 10^{-19} \text{ C} \right) \left( 6.80 \times 10^5 \text{ m/s} \right) \left( 8.70 \times 10^{-4} \text{ T} \right)} \right] = 19.7^\circ$$

74. **REASONING** At the center of the loop, the magnitude $B$ of the magnetic field due to the current $I$ in the long, straight wire is equal to the magnitude $B_{\text{loop}}$ of the magnetic field due to the current $I_{\text{loop}}$ in the circular wire loop. We know this because the vector sum of the two magnetic fields at that location is zero. We will determine the magnitude of the magnetic field due to the current in the circular loop from $B_{\text{loop}} = \frac{\mu_0 I_{\text{loop}}}{2R}$ (Equation 21.6, with $N = 1$), where $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space, and $R$ is the radius of the loop. The magnitude $B$ of the magnetic field of the long, straight wire is given by $B = \frac{\mu_0 I}{2\pi r}$ (Equation 21.5), where $r$ is the radial distance from the wire to the center of the loop. Because the wire is tangent to the loop, the radial distance $r$ is equal to the radius $R$ of the loop, and we have that

$$B = \frac{\mu_0 I}{2\pi R} \quad (1)$$

**SOLUTION** The two magnetic fields have equal magnitudes at the center of the circular loop, so $B_{\text{loop}} = \frac{\mu_0 I_{\text{loop}}}{2R}$ (Equation 21.6) and Equation (1) together yield

$$B_{\text{loop}} = \frac{\mu_0 I_{\text{loop}}}{2R} = \frac{\mu_0 I}{2\pi R} = B \quad (2)$$

Solving Equation (2) for $I_{\text{loop}}$, we obtain

$$\frac{\mu_0 I_{\text{loop}}}{2\pi R} = \frac{\mu_0 I}{2\pi R} \quad \text{or} \quad I_{\text{loop}} = \frac{I}{\pi} = 0.12 \text{ A} \quad \text{or} \quad I_{\text{loop}} = \frac{I}{\pi} = 0.038 \text{ A}$$

75. **SSM REASONING** According to Equation 21.4, the maximum torque is $\tau_{\text{max}} = N I A B$, where $N$ is the number of turns in the coil, $I$ is the current, $A = \pi r^2$ is the area of the circular coil, and $B$ is the magnitude of the magnetic field. Since the coil contains only one turn, the length $L$ of the wire is the circumference of the circle, so that $L = 2\pi r$ or $r = L/(2\pi)$. Since $N$, $I$, and $B$ are known we can solve for $L$.

**SOLUTION** According to Equation 21.4 and the fact that $r = L/(2\pi)$, we have

$$\tau_{\text{max}} = N I \pi r^2 B = N I \pi \frac{L}{2\pi} B$$
Solving this result for \( L \) gives

\[
L = \sqrt{\frac{4\pi \tau_{\text{max}}}{NIB}} = \sqrt{\frac{4\pi \times 4 \times 10^{-4}}{0.7 A \times 0.75 T}} = 0.062 \text{ m}
\]

76. **REASONING** The magnitude of the magnetic force acting on the particle is

\[ F = |q_0| vB \sin \theta \]  (Equation 21.1), where \( |q_0| \) and \( v \) are the charge magnitude and speed of the particle, respectively, \( B \) is the magnitude of the magnetic field, and \( \theta \) is the angle between the particle's velocity and the magnetic field. The magnetic field is produced by a very long, straight wire, so its value is given by Equation 21.5 as \( B = \mu_0 I / (2\pi r) \). By combining these two relations, we can determine the magnitude of the magnetic force.

**SOLUTION** The direction of the magnetic field \( \mathbf{B} \) produced by the current-carrying wire can be found by using Right-Hand Rule No. 2. At the location of the charge, this field points perpendicularly into the page, as shown in the drawing. Since the direction of the particle’s velocity is perpendicular to the magnetic field, \( \theta = 90.0^\circ \). Substituting \( B = \mu_0 I / (2\pi r) \) into

\[ F = |q_0| vB \sin \theta \]  gives

\[
F = |q_0| vB \sin \theta = |q_0| v \left( \frac{\mu_0 I}{2\pi r} \right) \sin \theta
\]

\[
= \left( 6.00 \times 10^{-6} \text{ C} \right) \left( 7.50 \times 10^4 \text{ m/s} \right) \left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) \left( 67.0 \text{ A} \right) \sin 90.0^\circ
\]

\[
= \frac{1.21 \times 10^{-4} \text{ N}}{2\pi \left( 5.00 \times 10^{-2} \text{ m} \right)}
\]

The direction of the magnetic force \( \mathbf{F} \) exerted on the particle can be determined by using Right-Hand Rule No. 1. This direction, which is shown in the drawing, is perpendicular to the wire and is directed away from it.

77. **REASONING** A maximum magnetic force is exerted on the wire by the field components that are perpendicular to the wire, and no magnetic force is exerted by field components that are parallel to the wire. Thus, the wire experiences a force only from the \( x \)- and \( y \)-components of the field. The \( z \)-component of the field may be ignored, since it is parallel to the wire. We can use the Pythagorean theorem to find the net field in the \( x, y \) plane. This net field, then, is perpendicular to the wire and makes an angle of \( \theta = 90^\circ \) with respect to the wire. Equation 21.3 can be used to calculate the magnitude of the magnetic force that this net field applies to the wire.
SOLUTION According to Equation 21.3, the magnetic force has a magnitude of \( F = ILB \sin \theta \), where \( I \) is the current, \( B \) is the magnitude of the magnetic field, \( L \) is the length of the wire, and \( \theta \) is the angle of the wire with respect to the field. Using the Pythagorean theorem, we find that the net field in the \( x, y \) plane is

\[
B = \sqrt{B_x^2 + B_y^2}
\]

Using this field in Equation 21.3, we calculate the magnitude of the magnetic force to be

\[
F = ILB \sin \theta = IL \sqrt{B_x^2 + B_y^2} \sin \theta
\]

\[
= 2.3 \text{ A} \cdot 0.25 \text{ m} \cdot 10 \text{ T} \cdot 0.15 \text{ T} \cdot \sin 90^\circ = 0.19 \text{ N}
\]

78. REASONING The current \( I \) is the rate of flow of charge and is \( I = \frac{\Delta q}{\Delta t} \) (Equation 20.1), where \( \Delta q \) is the amount of charge that flows in a time period \( \Delta t \). We can solve this equation for \( \Delta q \). The current is not given. However, since we are assuming that the bolt can be represented as a long, straight line of current, we can obtain the current from the information provided about the magnetic field \( B \). The field is \( B = \frac{\mu_0 I}{2\pi r} \) (Equation 21.5), where \( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \) is the permeability of free space and \( r \) is the perpendicular distance from the current.

SOLUTION Solving Equation 20.1 for the charge \( \Delta q \), we have

\[
I = \frac{\Delta q}{\Delta t} \quad \text{or} \quad \Delta q = I \Delta t
\]

Solving Equation 21.5 for the current \( I \), we have

\[
B = \frac{\mu_0 I}{2\pi r} \quad \text{or} \quad I = \frac{B 2\pi r}{\mu_0}
\]

Substituting this result for \( I \) into the expression for \( \Delta q \), we find that

\[
\Delta q = I \Delta t = \left( \frac{B 2\pi r}{\mu_0} \right) \Delta t = \left[ \frac{(8.0 \times 10^{-5} \text{ T}) 2\pi (27 \text{ m})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \right] (1.8 \times 10^{-3} \text{ s}) = 19 \text{ C}
\]

79. REASONING When a charge \( q_0 \) travels at a speed \( v \) and its velocity makes an angle \( \theta \) with respect to a magnetic field of magnitude \( B \), the magnetic force acting on the charge has a magnitude \( F \) that is given by \( F = |q_0| vB \sin \theta \) (Equation 21.1). We will solve this problem by applying this expression twice, first to the motion of the charge when it moves perpendicular to the field so that \( \theta = 90.0^\circ \) and then to the motion when \( \theta = 38^\circ \).
**SOLUTION** When the charge moves perpendicular to the field so that $\theta = 90.0^\circ$, Equation 21.1 indicates that

$$F_{90.0^\circ} = |q_0| vB \sin 90.0^\circ$$

When the charge moves so that $\theta = 38^\circ$, Equation 21.1 shows that

$$F_{38^\circ} = |q_0| vB \sin 38^\circ$$

Dividing the second expression by the first expression gives

$$\frac{F_{38^\circ}}{F_{90.0^\circ}} = \frac{|q_0| vB \sin 38^\circ}{|q_0| vB \sin 90.0^\circ}$$

$$\quad \quad \quad \quad F_{38^\circ} = F_{90.0^\circ} \left( \frac{\sin 38^\circ}{\sin 90.0^\circ} \right) = \left( 2.7 \times 10^{-3} \text{ N} \right) \left( \frac{\sin 38^\circ}{\sin 90.0^\circ} \right) = 1.7 \times 10^{-3} \text{ N}$$

80. **REASONING** Each of the four wires makes a contribution to the net magnetic field $B$ at the center of the square. The magnitude of each wire’s magnetic field is given by $B = \frac{\mu_0 I}{2\pi r}$ (Equation 21.5), where $I$ is the current in a wire, $r$ is the radial distance from the wire to the center of the square, and $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space. For all four wires, the radial distance $r$ is half of the length $s$ of one of the sides of the square ($r = \frac{1}{2}s$).

In order to add up the four magnetic fields, we must determine the direction of each one. Right-Hand Rule No. 2 predicts that, at the center of the square, the currents $I_1 = 3.9 \text{ A}$, $I_2 = 8.5 \text{ A}$, and $I_3 = 4.6 \text{ A}$ all produce fields pointing into the page, while the magnetic field $B$ of the unknown current $I$ points out of the page. Therefore, the magnitude $B_{\text{net}}$ of the net magnetic field is given by

$$B_{\text{net}} = B_1 + B_2 + B_3 - B$$

(1)

**SOLUTION** Applying $B = \frac{\mu_0 I}{2\pi r}$ (Equation 21.5) to the magnetic fields on the right side of Equation (1), we obtain

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r} + \frac{\mu_0 I_2}{2\pi r} + \frac{\mu_0 I_3}{2\pi r} - \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{2\pi r} (I_1 + I_2 + I_3 - I)$$

(2)

Solving Equation (2) for $I$ yields

$$\frac{2\pi r B_{\text{net}}}{\mu_0} = I_1 + I_2 + I_3 - I$$

or

$$I = I_1 + I_2 + I_3 - \frac{2\pi r B_{\text{net}}}{\mu_0}$$

(3)
The length $s$ of a side of the square is $s = 0.050$ m, so the radial distance $r$ from each wire to the center of the square is half as large: $r = \frac{1}{2}s = 0.025$ m. Therefore, Equation (3) yields

$$I = 3.9 \, \text{A} + 8.5 \, \text{A} + 4.6 \, \text{A} - \frac{2\pi (0.025 \, \text{m})(61 \times 10^{-6} \, \text{T})}{4\pi \times 10^{-7} \, \text{T} \cdot \text{m/A}} = 9.4 \, \text{A}$$

81. SSM REASONING From the discussion in Section 21.3, we know that when a charged particle moves perpendicular to a magnetic field, the trajectory of the particle is a circle. The drawing at the right shows a particle moving in the plane of the paper (the magnetic field is perpendicular to the paper). If the particle is moving initially through the coordinate origin and to the right (along the $+x$ axis), the subsequent circular path of the particle will intersect the $y$ axis at the greatest possible value, which is equal to twice the radius $r$ of the circle.

SOLUTION

a. From the drawing above, it can be seen that the largest value of $y$ is equal to the diameter $(2r)$ of the circle. When the particle passes through the coordinate origin its velocity must be parallel to the $+x$ axis. Thus, the angle is $\theta = 0^\circ$.

b. The maximum value of $y$ is twice the radius $r$ of the circle. According to Equation 21.2, the radius of the circular path is $r = \frac{mv}{(|q|B)}$. The maximum value $y_{\text{max}}$ is, therefore,

$$y_{\text{max}} = 2r = 2 \left( \frac{mv}{|q|B} \right) = 2 \left[ \frac{(3.8 \times 10^{-8} \, \text{kg})(44 \, \text{m/s})}{(7.3 \times 10^{-6} \, \text{C})(1.6 \, \text{T})} \right] = 0.29 \, \text{m}$$

82. REASONING

a. If the particle is stationary, only the electric field $E_x$ exerts a force on it; this force is $F_E = q E_x$ (Equation 18.2), where $q$ is the charge. A magnetic field does not exert a force on a stationary particle.
b. If the particle is moving along the \(+x\) axis, the electric field exerts a force on it. This force is the same as that in part (a) above. The magnetic field \(B_x\) does not exert a force on the particle, because the particle's velocity is parallel to the field. The magnetic field \(B_y\) does exert a force on the particle, because the particle's velocity is perpendicular to this field.

c. The particle experiences a force from each of the three fields. The electric field exerts a force on it, and this force is the same as that in part (a) above. It does not matter in which direction the particle travels, for the electric force is independent of the particle's velocity. The particle experiences magnetic forces from both \(B_x\) and \(B_y\). When the particle moves along the \(+z\) axis, its velocity is perpendicular to both \(B_x\) and \(B_y\), so each field exerts a force on the particle.

**SOLUTION**

a. The electric force exerted on the particle is \(F_E = qE_x\) (Equation 18.2), so

\[
F_E = qE_x = (5.60 \times 10^{-6} \text{ C})(245 \text{ N/C}) = 1.37 \times 10^{-3} \text{ N}
\]

where the plus sign indicates that the force points along the \(+x\) axis. The magnitude \(F\) of the magnetic force is given by Equation 21.1 as \(F = |q_0|vB\sin \theta\). Since \(v = 0 \text{ m/s}\), the magnetic forces exerted by \(B_x\) and \(B_y\) are zero:

\[
F_{B_x} = 0 \text{ N} \quad F_{B_y} = 0 \text{ N}
\]

b. The electric force is the same as that computed in part (a), because this force does not depend on the velocity of the particle: \(F_E = 1.37 \times 10^{-3} \text{ N}\), where the plus sign indicates that the force points along the \(+x\) axis.

Since the velocity of the particle and \(B_x\) are along the \(+x\) axis (\(\theta = 0^\circ\) in Equation 21.1), the magnitude of the magnetic force is

\[
F_{B_z} = |q_0|vB_x \sin \theta = |q_0|vB_x \sin 0^\circ = 0 \text{ N}
\]

The magnetic force exerted by the magnetic field \(B_y\) on the charge has a magnitude of

\[
F_{B_y} = |q_0|vB_y \sin \theta = (5.60 \times 10^{-6} \text{ C})(375 \text{ m/s})(1.40 \text{ T}) \sin 90.0^\circ = 2.94 \times 10^{-3} \text{ N}
\]

An application of Right-hand Rule No. 1 shows that the direction of the magnetic force is along the \(+z\) axis.
c. The electric force is the same as that computed in part (a), because this force does not depend on the velocity of the particle: \( F_e = 1.37 \times 10^{-3} \, \text{N} \), where the plus sign indicates that the force points along the \(+x \, \text{axis}\).

When the particle moves along the \(+z\) axis, the magnetic field \( B_x \) exerts a force on the charge that has a magnitude of

\[
F_{Bx} = |q_0| v B_x \sin \theta = \left( 5.60 \times 10^{-6} \, \text{C} \right) (375 \, \text{m/s}) (1.80 \, \text{T}) \sin 90.0^\circ = 3.78 \times 10^{-3} \, \text{N}
\]

An application of Right-hand Rule No. 1 shows that the direction of the magnetic force is along the \(+y \, \text{axis}\).

When the particle moves along the \(+z\) axis, the magnetic field \( B_y \) exerts a force on the charge that has a magnitude of

\[
F_{By} = |q_0| v B_y \sin \theta = \left( 5.60 \times 10^{-6} \, \text{C} \right) (375 \, \text{m/s}) (1.40 \, \text{T}) \sin 90.0^\circ = 2.94 \times 10^{-3} \, \text{N}
\]

An application of Right-hand Rule No. 1 shows that the direction of the magnetic force is along the \(-x \, \text{axis}\).

83. **REASONING** Two wires that are parallel and carry current in the same direction exert attractive magnetic forces on one another, as Section 21.7 discusses. This attraction between the wires causes the spring to compress. When compressed, the spring exerts an elastic restoring force on each wire, as Section 10.1 discusses. For each wire, this restoring force acts to push the wires apart and balances the magnetic force, thus keeping the separation between the wires from decreasing to zero. Equation 10.2 (without the minus sign) gives the magnitude of the restoring force as \( F_x = kx \), where \( k \) is the spring constant and \( x \) is the magnitude of the displacement of the spring from its unstrained length. By setting the magnitude of the magnetic force equal to the magnitude of the restoring force, we will be able to find the separation between the rods when the current is present.

**SOLUTION** According to Equation 21.3, the magnetic force has a magnitude of \( F = ILB \sin \theta \), where \( I \) is the current, \( B \) is the magnitude of the magnetic field, \( L \) is the length of the wire, and \( \theta \) is the angle of the wire with respect to the field. Using RHR-2 reveals that the magnetic field produced by either wire is perpendicular to the other wire, so that \( \theta = 90^\circ \) in Equation 21.3, which becomes \( F = ILB \). According to Equation 21.5 the magnitude of the magnetic field produced by a long straight wire is \( B = \mu_0 I/(2\pi r) \). Substituting this expression into Equation 21.3 gives the magnitude of the magnetic force as
\[ F = IL \frac{\mu_0 I^2 L}{2\pi r} \]

Equating this expression to the magnitude of the restoring force from the spring gives

\[ \frac{\mu_0 I^2 L}{2\pi r} = kx \]

Solving for the separation \( r \), we find

\[ r = \frac{\mu_0 I^2 L}{2\pi kx} = \frac{2\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{2\pi \times 50 \text{ N/m} \times 0.020 \text{ m}} = 0.030 \text{ m} \]

84. **REASONING** The turns of the coil are as close together as possible without overlapping, meaning that there are no gaps between the turns. Therefore, the width of a single turn is equal to the diameter \( D = 2r \) of the wire, where \( r \) is the wire's radius. The number \( N \) of turns, therefore, is \( N = L/(2r) \), where \( L \) is the length of the solenoid. The number of turns per unit length is \( n \), where

\[ n = \frac{N}{L} = \frac{L/(2r)}{L} = \frac{1}{2r} \quad (1) \]

The magnitude \( B \) of the magnetic field inside a solenoid with \( n \) turns per meter and a current \( I \) is given by \( B = \mu_0 n I \) (Equation 21.7), where \( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \) is the permeability of free space. We will use Ohm's law, \( R = \frac{V}{I} \) (Equation 20.2), to determine the current \( I \) in the solenoid when the battery (Emf = \( V \)) is connected to it. The resistance \( R \) of the silver wire will be found from \( R = \rho \frac{L}{A} \) (Equation 20.3), where \( \rho = 1.59 \times 10^{-8} \Omega \cdot \text{m} \) is the resistivity of silver (see Table 20.1), \( L \) is the length of the wire, and \( A \) is its cross-sectional area.

**SOLUTION** Substituting Equation (1) into \( B = \mu_0 n I \) (Equation 21.7) and solving for the radius \( r \) yields

\[ B = \mu_0 n I = \frac{\mu_0 I}{2r} \quad \text{or} \quad r = \frac{\mu_0 I}{2B} \quad (2) \]

Solving \( R = \frac{V}{I} \) (Equation 20.2) for \( I \), we obtain \( I = \frac{V}{R} \). Substituting this result into Equation (2), we find that

\[ r = \frac{\mu_0 I}{2B} = \frac{\mu_0 V}{2BR} \quad (3) \]

Substituting \( R = \rho \frac{L}{A} \) (Equation 20.3) into Equation (3), we obtain
\[ r = \frac{\mu_0 V}{2BR} = \frac{\mu_0 V}{2B \left( \frac{\rho L}{A} \right)} = \frac{\mu_0 V A}{2B \rho L} \tag{4} \]

The wire has a circular cross-section, so we may express the area \( A \) in Equation (4) as \( A = \pi r^2 \). Substituting this expression into Equation (4) and simplifying the resulting expression, we find that

\[ J = \frac{\mu_0 V \pi r^2}{2B \rho L} \quad \text{or} \quad 1 = \frac{\mu_0 V \pi r}{2B \rho L} \tag{5} \]

Solving Equation (5) for \( r \) yields

\[ r = \frac{2B \rho L}{\mu_0 V \pi} = \frac{2 \left( 6.48 \times 10^{-3} \text{ T} \right) \left( 1.59 \times 10^{-8} \text{ T} \cdot \text{m} \right) \left( 25.0 \text{ m} \right)}{\left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) \left( 3.00 \text{ V} \right) \pi} = 4.35 \times 10^{-4} \text{ m} \]

85. **SSM REASONING** The magnetic moment of the rotating charge can be found from the expression \( \text{Magnetic moment} = N I A \), as discussed in Section 21.6. For this situation, \( N = 1 \). Thus, we need to find the current and the area for the rotating charge. This can be done by resorting to first principles.

**SOLUTION** The current for the rotating charge is, by definition (see Equation 20.1), \( I = \frac{\Delta q}{\Delta t} \), where \( \Delta q \) is the amount of charge that passes by a given point during a time interval \( \Delta t \). Since the charge passes by once per revolution, we can find the current by dividing the total rotating charge by the period \( T \) of revolution.

\[ I = \frac{\Delta q}{T} = \frac{\Delta q}{2\pi / \omega} = \frac{\omega \Delta q}{2\pi} = \frac{\left( 150 \text{ rad/s} \right) \left( 4.0 \times 10^{-6} \text{ C} \right)}{2\pi} = 9.5 \times 10^{-5} \text{ A} \]

The area of the rotating charge is \( A = \pi r^2 = \pi (0.20 \text{ m})^2 = 0.13 \text{ m}^2 \)

Therefore, the magnetic moment is

\[ \text{Magnetic moment} = N I A = \left( 1 \right) \left( 9.5 \times 10^{-5} \text{ A} \right) \left( 0.13 \text{ m}^2 \right) = 1.2 \times 10^{-5} \text{ A} \cdot \text{m}^2 \]
CHAPTER 22 | ELECTROMAGNETIC INDUCTION

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. 3.5 m/s

2. (e) The work done by the hand equals the energy dissipated in the bulb. The energy dissipated in the bulb equals the power used by the bulb times the time. Since the time is the same in each case, more work is done when the power used is greater. The power, however, is the voltage squared divided by the resistance of the bulb, according to Equation 20.6c, so that a smaller resistance corresponds to a greater power. Thus, more work is done when the resistance of the bulb is smaller.

3. (c) The magnetic flux $\Phi$ that passes through a surface is $\Phi = BA \cos \phi$ (Equation 22.2), where $B$ is the magnitude of the magnetic field, $A$ is the area of the surface, and $\phi$ is the angle between the field and the normal to the surface. Knowing $\Phi$ and $A$, we can calculate $B \cos \phi = \Phi / A$, which is the component of the field parallel to the normal or perpendicular to the surface.

4. (b) The magnetic flux $\Phi$ that passes through a surface is $\Phi = BA \cos \phi$ (Equation 22.2), where $B$ is the magnitude of the magnetic field, $A$ is the area of the surface, and $\phi$ is the angle between the field and the normal to the surface. It has the greatest value when the field strikes the surface perpendicularly ($\phi = 0^\circ$) and a value of zero when the field is parallel to a surface ($\phi = 90^\circ$). The field is more nearly perpendicular to face 1 ($\phi = 20^\circ$) than to face 3 ($\phi = 70^\circ$) and is parallel to face 2.

5. (d) Faraday’s law of electromagnetic induction states that the average emf $\xi$ induced in a coil of $N$ loops is $\xi = -N(\Delta \Phi / \Delta t)$ (Equation 22.3), where $\Delta \Phi$ is the change in magnetic flux through one loop and $\Delta t$ is the time interval during which the change occurs. Reducing the time interval $\Delta t$ during which the field magnitude increases means that the rate of change of the flux will increase, which will increase (not reduce) the induced emf.

6. 3.2 V

7. (c) According to Faraday’s law, the magnitude of the induced emf is the magnitude of the change in magnetic flux divided by the time interval over which the change occurs (see Equation 22.3). In each case the field is perpendicular to the coil, and the initial flux is zero since the coil is outside the field region. Therefore, the changes in flux are as follows: $\Delta \Phi_A = BL^2$, $\Delta \Phi_B = \Delta \Phi_C = B2L^2$ (see Equation 22.2). The corresponding time intervals are
\[ \Delta t_A = \Delta t_B = \Delta t, \quad \Delta t_C = 2\Delta t. \] Dividing gives the following results for the magnitudes of the emfs: \[ |\xi_A| = \frac{BL^2}{\Delta t}, \quad |\xi_B| = \frac{B2L^2}{\Delta t}, \quad |\xi_C| = \frac{B2L^2}{2\Delta t} = \frac{BL^2}{\Delta t}. \]

8. (a) An induced current appears only when there is an induced emf to drive it around the loop. According to Faraday's law, an induced emf exists only when the magnetic flux through the loop changes as time passes. Here, however, there is no magnetic flux through the loop. The magnetic field lines produced by the current are circular and centered on the wire, with the planes of the circles perpendicular to the wire. Therefore, the magnetic field is always parallel to the plane of the loop as the loop falls and never penetrates the loop. In other words, no magnetic flux passes through the loop. No magnetic flux, no induced emf, no induced current.

9. (d) When the switch is closed, current begins to flow counterclockwise in the larger coil, and the field that it creates appears inside the smaller coil. Using RHR-2 reveals that this field points out of the screen toward you. According to Lenz's law, the induced current in the smaller coil flows in such a direction that it creates an induced field that opposes the growth of the field from the larger coil. Thus, the induced field must point into the screen away from you. Using RHR-2 reveals that the induced current must, then, flow clockwise. The induced current exists only for the short period following the closing of the switch, when the field from the larger coil is growing from zero to its equilibrium value. Once the field from the larger coil reaches its equilibrium value and ceases to change, the induced current in the smaller coil becomes zero.

10. (b) The peak emf is proportional to the area \( A \) of the coil, according to Equation 22.4. Thus, we need to consider the areas of the coils. The length of the wire is \( L \) and is the same for each of the coil shapes. For the circle, the circumference is \( 2\pi r = L \), so that the area is \[ A_{\text{circle}} = \pi r^2 = \pi \left( \frac{L}{2\pi} \right)^2 = \frac{L^2}{4\pi}. \] For the square, the area is \[ A_{\text{square}} = \left( \frac{L}{4} \right)^2 = \frac{L^2}{16}. \] For the rectangle, the perimeter is \( 2(D + 2D) = L \), so that the area is \[ A_{\text{rectangle}} = \left( \frac{L}{6} \right) \left( \frac{2L}{6} \right) = \frac{L^2}{18}. \] The circle has the largest area, while the rectangle has the smallest area, corresponding to answer b.

11. 5.3 cm

12. (d) The back emf is proportional to the motor speed, so it decreases when the speed decreases. The current \( I \) drawn by the motor is given by Equation 22.5 as \[ I = \frac{V - \xi}{R}, \] where \( V \) is the voltage at the socket, \( \xi \) is the back emf, and \( R \) is the resistance of the motor coil. As \( \xi \) decreases, \( I \) increases.
13. (c) According to Equation 22.7, the mutual inductance is $M = \frac{-\dot{\phi}_S}{\Delta I_p}$. If the time interval is cut in half and the change in the primary current is doubled, while the induced emf remains the same, the mutual inductance must be reduced by a factor of four.

14. (b) The energy stored in an inductor is given by Equation 22.10 as Energy = $\frac{1}{2} L I^2$. Since the two inductors store the same amount of energy, we have $\frac{1}{2} L_1 I_1^2 = \frac{1}{2} L_2 I_2^2$. Thus, 

$$\frac{I_1}{I_2} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{L_2}{2 L_1}} = \sqrt{2} = 1.414.$$

15. (e) According to Equation 22.8, we have $N = \frac{LI}{\Phi}$. Since $\Phi$ is the same for each coil, the number of turns is proportional to the product $LI$ of the inductance and the current. For the coils specified in the table, this product is $(LI)_A = L_0 I_0$, $(LI)_B = L_0 I_0^2$, $(LI)_C = 4L_0 I_0$.

16. (c) The current in the primary is proportional to the current in the secondary according to Equation 22.13: $I_p = I S_N S / N_p$. The current in the secondary is the secondary voltage divided by the resistance, according to Ohm's law. Thus, when the resistance increases, the current in the secondary decreases and so does the current in the primary. The wall socket delivers to the primary the same power that the secondary delivers to the resistance, assuming that no power is lost within the transformer. The power delivered to the resistance is given by Equation 20.15c as the square of the secondary voltage divided by the resistance. When the resistance increases, the power decreases. Hence, the power delivered to the primary by the wall socket also decreases.

17. 0.31 W

18. (a) The current in resistor 2 (without the transformer) is the same as the current in resistor 1 (with the transformer). In either event, the current $I$ is $I = V/R$, where $V$ is the voltage across the resistance $R$. Since the transformer is a step-up transformer, the voltage applied across resistor 2 is smaller than the voltage applied across resistor 1. The smaller voltage across resistor 2 can lead to the same current as does the greater voltage across resistor 1 only if $R_2$ is less than $R_1$. 
CHAPTER 22 | ELECTROMAGNETIC INDUCTION

PROBLEMS

1. **REASONING** During its fall, both the length of the bar and its velocity \( v \) are perpendicular to the horizontal component \( B_h \) of the earth's magnetic field (see the drawing for an overhead view). Therefore, the emf \( \xi \) induced across the length \( L \) of the rod is given by \( \xi = vB_hL \) (Equation 22.1), where \( v \) is the speed of the rod. We will use Equation 22.1 to determine the magnitude \( B_h \) of the horizontal component of the earth’s magnetic field, and Right-Hand Rule No.1 from Section 21.2 to determine which end of the rod is positive.

**SOLUTION**

a. Solving \( \xi = vB_hL \) (Equation 22.1) for \( B_h \), we find that

\[
B_h = \frac{\xi}{vL} = \frac{6.5 \times 10^{-4} \text{ V}}{(22 \text{ m/s})(0.80 \text{ m})} = 3.7 \times 10^{-5} \text{ T}
\]

b. Consider a hypothetical positive charge that is free to move inside the falling rod. The bar is falling downward, carrying the positive charge with it, so that the velocity \( v \) of the charge is downward. In the drawing, which shows the situation as seen from above, downward is into the page. Applying Right-Hand Rule No. 1 to the vectors \( v \) and \( B_h \), the magnetic force \( F \) on the charge points to the east. Therefore, positive charges in the rod would accelerate to the east, and negative charges would accelerate to the west. As a result, the east end of the rod acquires a positive charge.

2. **REASONING** We can treat the car as if it were a straight rod of length \( L = 2.0 \text{ m} \) between the driver's side and the passenger's side. This “rod” and the vertically downward component of the earth’s magnetic field are perpendicular. The emf \( \xi \) induced between the ends of this moving “rod” is \( \xi = vBL \) (Equation 22.1), where \( v = 25 \text{ m/s} \) is the speed of the car, and \( B = 4.8 \times 10^{-5} \text{ T} \) is the vertically downward component of the earth’s magnetic field. To determine which side of the car is positive, we can use right-hand rule no. 1 (See Section 21.2), which will tell us the end of the rod at which positive charge will accumulate.
**SOLUTION**
a. Using Equation 22.1, we find that the emf is

\[ \xi = vBL = (25 \text{ m/s}) (4.8 \times 10^{-5} \text{ T}) (2.0 \text{ m}) = 2.4 \times 10^{-3} \text{ V} \]

b. Right-hand rule no. 1 shows that positive charge accumulates on the **driver's side**.

---

3. **REASONING AND SOLUTION** The motional emf \( \xi \) generated by a conductor moving perpendicular to a magnetic field is given by Equation 22.1 as \( \xi = vBL \), where \( v \) and \( L \) are the speed and length, respectively, of the conductor, and \( B \) is the magnitude of the magnetic field. The emf would have been

\[ \xi = vBL = (7.6 \times 10^3 \text{ m/s}) (5.1 \times 10^{-5} \text{ T}) (2.0 \times 10^4 \text{ m}) = 7800 \text{ V} \]

---

4. **REASONING** The situation in the drawing given with the problem statement is analogous to that in Figure 22.4b in the text. The blood flowing at a speed \( v \) corresponds to the moving rod, and the diameter of the blood vessel corresponds to the length \( L \) of the rod in the figure. The magnitude of the magnetic field is \( B \), and the measured voltage is the emf \( \xi \) induced by the motion. Thus, we can apply \( \xi = BVL \) (Equation 22.1).

**SOLUTION** Using Equation 22.1, we find that

\[ \xi = BVL = (0.60 \text{ T})(0.30 \text{ m/s})(5.6 \times 10^{-3} \text{ m}) = 1.0 \times 10^{-3} \text{ V} \]

---

5. **SSM** **REASONING AND SOLUTION** For the three rods in the drawing in the text, we have the following:

**Rod A:** The motional emf is **zero**, because the velocity of the rod is parallel to the direction of the magnetic field, and the charges do not experience a magnetic force.

**Rod B:** The motional emf \( \xi \) is, according to Equation 22.1,

\[ \xi = vBL = (2.7 \text{ m/s})(0.45 \text{ T})(1.3 \text{ m}) = 1.6 \text{ V} \]

The positive end of Rod B is **end 2**.

**Rod C:** The motional emf is **zero**, because the magnetic force \( F \) on each charge is directed perpendicular to the length of the rod. For the ends of the rod to become charged, the magnetic force must be directed parallel to the length of the rod.
6. **REASONING**
   a. The motional emf generated by the moving metal rod depends only on its speed, its length, and the magnitude of the magnetic field (see Equation 22.1). The motional emf does not depend on the resistance in the circuit. Therefore, the emfs for the circuits are the same.

   b. According to Equation 20.2, the current \( I \) is equal to the emf divided by the resistance \( R \) of the circuit. Since the emfs in the two circuits are the same, the circuit with the smaller resistance has the larger current. Since circuit 1 has one-half the resistance of circuit 2, the current in circuit 1 is twice as large.

   c. The power \( P \) is \( P = \frac{\xi^2}{R} \) (Equation 20.6c), where \( \xi \) is the emf (or voltage) and \( R \) is the resistance. The emf produced by the moving bar is directly proportional to its speed (see Equation 22.1). Thus, the bar in circuit 1 produces twice the emf, since it's moving twice as fast. Moreover, the resistance in circuit 1 is half that in circuit 2. As a result, the power delivered to the bulb in circuit 1 is \( 2^2 / \left(\frac{1}{2}\right) = 8 \) times greater than in circuit 2.

**SOLUTION**
   a. The ratio of the emfs is, according to Equation 22.1
   
   \[
   \frac{\xi_1}{\xi_2} = \frac{vBL}{v'BL} = \frac{1}{1}
   \]

   b. Equation 20.2 states that the current is equal to the emf divided by the resistance. The ratio of the currents is
   
   \[
   \frac{I_1}{I_2} = \frac{\xi_1/R_1}{\xi_2/R_2} = \frac{R_2}{R_1} = \frac{110 \Omega}{55 \Omega} = \frac{2}{1}
   \]

   c. The power, according to Equation 20.6c, is \( P = \frac{\xi^2}{R} \). The motional emf is given by Equation 22.1 as \( \xi = vBL \). The ratio of the powers is
   
   \[
   \frac{P_1}{P_2} = \frac{\frac{\xi_1^2}{R_1}}{\frac{\xi_2^2}{R_2}} = \left( \frac{\xi_1}{\xi_2} \right)^2 \left( \frac{R_2}{R_1} \right) = \left( \frac{v_1BL}{v_2BL} \right)^2 \left( \frac{R_2}{R_1} \right)
   \]
   
   \[
   = \left( \frac{v_1}{v_2} \right)^2 \left( \frac{R_2}{R_1} \right) = \left( \frac{2v_2}{v_2} \right)^2 \left( \frac{R_2}{\frac{1}{2}R_2} \right) = 8
   \]

7. **REASONING** Once the switch is closed, there is a current in the rod. The magnetic field applies a force to this current and accelerates the rod to the right. As the rod begins to move, however, a motional emf appears between the ends of the rod. This motional emf depends on the speed of the rod and increases as the speed increases. Equally important is the fact that the motional emf opposes the emf of the battery. The net emf causing the current in the
rods is the algebraic sum of the two emf contributions. Thus, as the speed of the rod increases and the motional emf increases, the net emf decreases. As the net emf decreases, the current in the rod decreases and so does the force that field applies to the current. Eventually, the speed reaches the point when the motional emf has the same magnitude as the battery emf, and the net emf becomes zero. At this point, there is no longer a net force acting on the rod and the speed remains constant from this point onward, according to Newton’s second law. This maximum speed can be determined by using Equation 22.1 for the motional emf, with a value of the motional emf that equals the battery emf.

**SOLUTION** Using Equation 22.1 \((\xi = vBL)\) and a value of 3.0 V for the emf, we find that the maximum speed of the rod is

\[
v = \frac{\xi}{BL} = \frac{3.0 \text{ V}}{(0.60 \text{ T})(0.20 \text{ m})} = 25 \text{ m/s}
\]

8. **REASONING** The average power \(\bar{P}\) delivered by the hand is given by \(\bar{P} = \frac{W}{t}\), where \(W\) is the work done by the hand and \(t\) is the time interval during which the work is done. The work done by the hand is equal to the product of the magnitude \(F_{\text{hand}}\) of the force exerted by the hand, the magnitude \(x\) of the rod’s displacement, and the cosine of the angle between the force and the displacement.

Since the rod moves to the right at a constant speed, it has no acceleration and is, therefore, in equilibrium. Thus, the force exerted by the hand must be equal to the magnitude \(F\) of the magnetic force that the current exerts on the rod. The magnitude of the magnetic force is given by \(F = ILB \sin \theta\) (Equation 21.3), where \(I\) is the current, \(L\) is the length of the moving rod, \(B\) is the magnitude of the magnetic field, and \(\theta\) is the angle between the direction of the current and that of the magnetic field.

**SOLUTION** The average power \(\bar{P}\) delivered by the hand is

\[
\bar{P} = \frac{W}{t} \quad (6.10a)
\]

The work \(W\) done by the hand in Figure 22.5 is given by \(W = F_{\text{hand}}x \cos \theta'\) (Equation 6.1). In this equation \(F_{\text{hand}}\) is the magnitude of the force that the hand exerts on the rod, \(x\) is the magnitude of the rod’s displacement, and \(\theta'\) is the angle between the force and the displacement. The force and displacement point in the same direction, so \(\theta' = 0^\circ\). Since the magnitude of the force exerted by the hand equals the magnitude \(F\) of the magnetic force, \(F_{\text{hand}} = F\). Substituting \(W = F_{\text{hand}}x \cos 0^\circ\) into Equation 6.10a and using the fact that \(F_{\text{hand}} = F\), we have that

\[
\bar{P} = \frac{W}{t} = \frac{F_{\text{hand}}x \cos 0^\circ}{t} = \frac{Fx \cos 0^\circ}{t}
\]

The magnitude \(F\) of the magnetic force is given by \(F = ILB \sin \theta\) (Equation 21.3). In this case, the current and magnetic field are perpendicular to each other, so \(\theta = 90^\circ\) (see Figure 22.5). Substituting this expression for \(F\) into Equation 1 gives
\[ \bar{P} = \frac{F \cos 0^\circ}{t} = \frac{(ILB \sin 90^\circ) \cos 0^\circ}{t} \]

The term \( \frac{F}{t} \) in Equation 2 is the speed \( v \) of the rod. Thus, the average power delivered by the rod is

\[ \bar{P} = \frac{(ILB \sin 90^\circ) \cos 0^\circ}{t} = (ILB \sin 90^\circ) v \cos 0^\circ \]

\[ = (0.040 \, \text{A})(0.90 \, \text{m})(1.2 \, \text{T}) \sin 90^\circ(3.5 \, \text{m/s}) \cos 0^\circ = 0.15 \, \text{W} \]

9. **SSM REASONING** The minimum length \( d \) of the rails is the speed \( v \) of the rod times the time \( t \), or \( d = vt \). We can obtain the speed from the expression for the motional emf given in Equation 22.1. Solving this equation for the speed gives \( v = \frac{\xi}{BL} \), where \( \xi \) is the motional emf, \( B \) is the magnitude of the magnetic field, and \( L \) is the length of the rod. Thus, the length of the rails is \( d = vt = \left( \frac{\xi}{BL} \right) t \). While we have no value for the motional emf, we do know that the bulb dissipates a power of \( P = 60.0 \, \text{W} \), and has a resistance of \( R = 240 \, \Omega \). Power is related to the emf and the resistance according to \( P = \frac{\xi^2}{R} \) (Equation 20.6c), which can be solved to show that \( \xi = \sqrt{PR} \). Substituting this expression into the equation for \( d \) gives

\[ d = \left( \frac{\xi}{BL} \right) t = \left( \frac{\sqrt{PR}}{BL} \right) t \]

**SOLUTION** Using the above expression for the minimum necessary length of the rails, we find that

\[ d = \left( \frac{\sqrt{PR}}{BL} \right) t = \frac{\sqrt{(60.0 \, \text{W})(240 \, \Omega)}}{(0.40 \, \text{T})(0.60 \, \text{m})} (0.50 \, \text{s}) = 250 \, \text{m} \]

10. **REASONING AND SOLUTION**

a. Newton's second law gives the magnetic retarding force to be

\[ F = mg = IBL \]

Now the current, \( I, \) is

\[ I = \frac{\xi}{R} = \frac{vBL}{R} \]

So

\[ m = \frac{v(BL)^2}{Rg} = \frac{(4.0 \, \text{m/s})(0.50 \, \text{T})^2 (1.3 \, \text{m})^2}{(0.75 \, \Omega)(9.80 \, \text{m/s}^2)} = 0.23 \, \text{kg} \]
b. The change in height in a time $\Delta t$ is $\Delta h = -v\Delta t$. The change in gravitational potential energy is

$$\Delta PE = mg\Delta h = -mgv\Delta t = -(0.23 \text{ kg})(9.80 \text{ m/s}^2)(4.0 \text{ m/s})(0.20 \text{ s}) = -1.8 \text{ J}$$

c. The energy dissipated in the resistor is the amount by which the gravitational potential energy decreases or $1.8 \text{ J}$.

---

11. **REASONING** The definition of magnetic flux $\Phi$ is $\Phi = BA\cos\phi$ (Equation 22.2), where $B$ is the magnitude of the magnetic field, $A$ is the area of the surface, and $\phi$ is the angle between the magnetic field vector and the normal to the surface. The values of $B$ and $A$ are the same for each of the surfaces, while the values for the angle $\phi$ are different. The $z$ axis is the normal to the surface lying in the $x, y$ plane, so that $\phi_{xz} = 35^\circ$. The $y$ axis is the normal to the surface lying in the $x, z$ plane, so that $\phi_{xy} = 55^\circ$. We can apply the definition of the flux to obtain the desired ratio directly.

**SOLUTION** Using Equation 22.2, we find that

$$\frac{\Phi_{xz}}{\Phi_{xy}} = \frac{BA\cos\phi_{xz}}{BA\cos\phi_{xy}} = \frac{\cos 55^\circ}{\cos 35^\circ} = 0.70$$

---

12. **REASONING**

a. The magnetic flux $\Phi$ though a surface of area $A$ due to a uniform magnetic field of magnitude $B$ is given by $\Phi = BA\cos\phi$ (Equation 22.2) where $\phi$ is the angle between the direction of the magnetic field and the normal to the surface (see the drawing, which shows an edge-on view of the situation). The magnetic field $B$ is horizontal, and the surface makes an angle of $12^\circ$ with the horizontal, so the normal to the surface is $\phi = 90.0^\circ - 12^\circ = 78^\circ$. We will use Equation 22.2 to determine the surface area $A$.

b. When exposed to a uniform magnetic field, the magnetic flux through a flat surface is greatest when the surface is perpendicular to the direction of the magnetic field. When this occurs, the normal to the surface is parallel to the direction of the magnetic field, and the angle $\phi$ is zero. Therefore, to find the smallest surface area that has same amount of magnetic flux passing through it as the surface in part (a), we will take $\phi = 0.0^\circ$ in Equation 22.2.
**SOLUTION**

a. Solving \( \Phi = BA \cos \phi \) (Equation 22.2) for \( A \) yields \( A = \frac{\Phi}{B \cos \phi} \). Substituting \( \phi = 78^\circ \), we find that

\[
A = \frac{\Phi}{B \cos \phi} = \frac{8.4 \times 10^{-3} \text{ Wb}}{(0.47 \text{ T}) \cos 78^\circ} = 0.086 \text{ m}^2
\]

b. From \( A = \frac{\Phi}{B \cos \phi} \), with \( \phi = 0.0^\circ \), the minimum possible area of the second surface is

\[
A = \frac{8.4 \times 10^{-3} \text{ Wb}}{(0.47 \text{ T}) \cos 0.0^\circ} = 0.018 \text{ m}^2
\]

13. **REASONING** The general expression for the magnetic flux through an area \( A \) is given by Equation 22.2: \( \Phi = BA \cos \phi \), where \( B \) is the magnitude of the magnetic field and \( \phi \) is the angle of inclination of the magnetic field \( B \) with respect to the normal to the area.

The magnetic flux through the door is a maximum when the magnetic field lines are perpendicular to the door and \( \phi_1 = 0.0^\circ \) so that \( \Phi_1 = \Phi_{\text{max}} = BA(\cos 0.0^\circ) = BA \).

**SOLUTION** When the door rotates through an angle \( \phi_2 \), the magnetic flux that passes through the door decreases from its maximum value to one-third of its maximum value. Therefore, \( \Phi_2 = \frac{1}{3} \Phi_{\text{max}} \), and we have

\[
\Phi_2 = BA \cos \phi_2 = \frac{1}{3} BA \quad \text{or} \quad \cos \phi_2 = \frac{1}{3} \quad \text{or} \quad \phi_2 = \cos^{-1} \left( \frac{1}{3} \right) = 70.5^\circ
\]

14. **REASONING** At any given moment, the flux \( \Phi \) that passes through the loop is given by \( \Phi = BA \cos \phi \) (Equation 22.2), where \( B \) is the magnitude of the magnetic field, \( A \) is the area of the loop, and \( \phi = 0^\circ \) is the angle between the normal to the loop and the direction of the magnetic field (both directed into the page). As the handle turns, the area \( A \) of the loop changes, causing a change \( \Delta \Phi \) in the flux passing through the loop. We can think of the loop as being divided into a rectangular portion and a semicircular portion. Initially, the area \( A_0 \) of the loop is equal to the rectangular area \( A_{\text{rec}} \) plus the area \( A_{\text{semi}} = \frac{1}{2} \pi r^2 \) of the semicircle, where \( r \) is the radius of the semicircle: \( A_0 = A_{\text{rec}} + \frac{1}{2} \pi r^2 \). After half a revolution, the semicircle is once again within the plane of the loop, but now as a reduction of the area of the rectangular portion. Therefore, the final area \( A \) of the loop is equal to the area of the rectangular portion minus the area of the semicircle: \( A = A_{\text{rec}} - \frac{1}{2} \pi r^2 \).

**SOLUTION** The change \( \Delta \Phi \) in the flux that passes through the loop is the difference between the final flux \( \Phi = BA \cos \phi \) (Equation 22.2) and the initial flux \( \Phi_0 = BA_0 \cos \phi \):
\[ \Delta \Phi = \Phi - \Phi_0 = BA \cos \phi - BA_0 \cos \phi = B \cos \phi (A - A_0) \]  \hspace{1cm} (1)

Substituting \( A_0 = A_{\text{rec}} + \frac{1}{2} \pi r^2 \) and \( A = A_{\text{rec}} - \frac{1}{2} \pi r^2 \) into Equation (1) yields

\[ \Delta \Phi = B \cos \phi (A - A_0) = B \cos \phi \left[ \left( A_{\text{rec}} - \frac{1}{2} \pi r^2 \right) - \left( A_{\text{rec}} + \frac{1}{2} \pi r^2 \right) \right] = -\pi r^2 B \cos \phi \]

Therefore the change in the flux passing through the loop during half a revolution of the semicircle is

\[ \Delta \Phi = -\pi r^2 B \cos \phi = -\pi (0.20 \, \text{m})^2 (0.75 \, \text{T}) \cos 0^\circ = -0.094 \, \text{Wb} \]

15. **REASONING** According to Equation 22.2, the magnetic flux \( \Phi \) is the product of the magnitude \( B \) of the magnetic field, the area \( A \) of the surface, and the cosine of the angle \( \phi \) between the direction of the magnetic field and the normal to the surface. The area of a circular surface is \( A = \pi r^2 \), where \( r \) is the radius.

**SOLUTION** The magnetic flux \( \Phi \) through the surface is

\[ \Phi = BA \cos \phi = B \left( \pi r^2 \right) \cos \phi = (0.078 \, \text{T}) \pi (0.10 \, \text{m})^2 \cos 25^\circ = 2.2 \times 10^{-3} \, \text{Wb} \]

16. **REASONING** The magnetic flux \( \Phi \) that passes through a flat single-turn loop of wire is \( \Phi = BA \cos \phi \) (Equation 22.2), where \( B \) is the magnitude of the magnetic field (the same for each loop since the field is uniform), \( A \) is the area of the loop, and \( \phi \) is the angle between the magnetic field and the normal to the plane of the loop. Since both the square and the circle are perpendicular to the field, we know that \( \phi = 0^\circ \) for both loops. We will apply Equation 22.2 to both the square and the circle.

**SOLUTION** Using \( L \) to denote the length of each side of the square and \( R \) to denote the radius of the circle and recognizing that the areas of the square and circle are, respectively, \( L^2 \) and \( \pi R^2 \), we have from Equation 22.2 that

\[ \Phi_{\text{square}} = BA_{\text{square}} \cos 0^\circ = BL^2 \hspace{1cm} \text{and} \hspace{1cm} \Phi_{\text{circle}} = BA_{\text{circle}} \cos 0^\circ = B \pi R^2 \]

Dividing the right-hand equation by the left-hand equation, we obtain

\[ \frac{\Phi_{\text{circle}}}{\Phi_{\text{square}}} = \frac{B \pi R^2}{BL^2} = \frac{\pi R^2}{L^2} \]

We do have a value for either \( L \) or \( R \), but we do know that the square and the circle both contain the same length of wire. Therefore, it follows that

\[ \frac{4L}{\text{Length of wire in square}} = \frac{2\pi R}{\text{Circumference of circle}} \hspace{1cm} \text{or} \hspace{1cm} R = \frac{2L}{\pi} \]
Substituting this result for the radius of the circle into Equation (1) gives

\[
\frac{\Phi_{\text{circle}}}{\Phi_{\text{square}}} = \frac{\pi R^2}{L^2} = \frac{\pi (2L/\pi)^2}{L^2} = \frac{4}{\pi}
\]

\[
\Phi_{\text{circle}} = \frac{4}{\pi} \Phi_{\text{square}} = \frac{4}{\pi} (7.0 \times 10^{-3} \text{ Wb}) = 8.9 \times 10^{-3} \text{ Wb}
\]

17. **REASONING** The general expression for the magnetic flux through an area \( A \) is given by Equation 22.2: \( \Phi = BA \cos \phi \), where \( B \) is the magnitude of the magnetic field and \( \phi \) is the angle of inclination of the magnetic field \( B \) with respect to the normal to the surface.

**SOLUTION** Since the magnetic field \( B \) is parallel to the surface for the triangular ends and the bottom surface, the flux through each of these three surfaces is \([0 \text{ Wb}]\).

The flux through the 1.2 m by 0.30 m face is

\[
\Phi = BA \cos \phi = (0.25 \text{ T})(1.2 \text{ m})(0.30 \text{ m}) \cos 0.0^\circ = 0.090 \text{ Wb}
\]

For the 1.2 m by 0.50 m side, the area makes an angle \( \phi \) with the magnetic field \( B \), where

\[
\phi = 90^\circ - \tan^{-1}\left(\frac{0.30 \text{ m}}{0.40 \text{ m}}\right) = 53^\circ
\]

Therefore,

\[
\Phi = BA \cos \phi = (0.25 \text{ T})(1.2 \text{ m})(0.50 \text{ m}) \cos 53^\circ = 0.090 \text{ Wb}
\]

18. **REASONING** An emf is induced during the first and third intervals, because the magnetic field is changing in time. The time interval is the same (3.0 s) for the two cases. However, the magnitude of the field changes more during the first interval. Therefore, the magnetic flux is changing at a greater rate in that interval, which means that the magnitude of the induced emf is greatest during the first interval.

The induced emf is zero during the second interval, 3.0 – 6.0 s. According to Faraday’s law of electromagnetic induction, Equation 22.3, an induced emf arises only when the magnetic flux changes. During this interval, the magnetic field, the area of the loop, and the orientation of the field relative to the loop are constant. Thus, the magnetic flux does not change, so there is no induced emf.

During the first interval the magnetic field in increasing with time. During the third interval, the field is decreasing with time. As a result, the induced emfs will have opposite polarities during these intervals. If the direction of the induced current is clockwise during the first interval, it will be counterclockwise during the third interval.

**SOLUTION**

a. The induced emf is given by Equations 22.3 and 22.3:
0–3.0 s:
\[\xi = -N \frac{\Delta \Phi}{\Delta t} = -N \left( \frac{BA \cos \phi - B_0 A \cos \phi}{t - t_0} \right)\]
\[- = -NA \cos \phi \left( \frac{B - B_0}{t - t_0} \right) = -(50)(0.15 \text{ m}^2)(\cos 0^\circ) \left( \frac{0.40 \text{ T} - 0 \text{ T}}{3.0 \text{ s} - 0 \text{ s}} \right) = -1.0 \text{ V}\]

3.0–6.0 s:
\[\xi = -NA \cos \phi \left( \frac{B - B_0}{t - t_0} \right) = -(50)(0.15 \text{ m}^2)(\cos 0^\circ) \left( \frac{0.40 \text{ T} - 0.40 \text{ T}}{6.0 \text{ s} - 3.0 \text{ s}} \right) = 0 \text{ V}\]

6.0–9.0 s:
\[\xi = -NA \cos \phi \left( \frac{B - B_0}{t - t_0} \right) = -(50)(0.15 \text{ m}^2)(\cos 0^\circ) \left( \frac{0.20 \text{ T} - 0.40 \text{ T}}{9.0 \text{ s} - 6.0 \text{ s}} \right) = +0.50 \text{ V}\]

b. The induced current is given by Equation 20.2 as \(I = \xi/R\).

0–3.0 s:
\[I = \frac{\xi}{R} = \frac{-1.0 \text{ V}}{0.50 \Omega} = -2.0 \text{ A}\]

6.0–9.0 s:
\[I = \frac{\xi}{R} = \frac{+0.50 \text{ V}}{0.50 \Omega} = +1.0 \text{ A}\]

As expected, the currents are in opposite directions.

19. **REASONING** The magnitude \(|\xi|\) of the emf induced in the loop can be found using Faraday's law of electromagnetic induction:

\[|\xi| = \left| -N \frac{\Phi - \Phi_0}{t - t_0} \right| \quad (22.3)\]

where \(N\) is the number of turns, \(\Phi\) and \(\Phi_0\) are, respectively, the final and initial fluxes, and \(t - t_0\) is the elapsed time. The magnetic flux is given by \(\Phi = BA \cos \phi\) (Equation 22.2), where \(B\) is the magnitude of the magnetic field, \(A\) is the area of the surface, and \(\phi\) is the angle between the direction of the magnetic field and the normal to the surface.

**SOLUTION** Setting \(N = 1\) since there is only one turn, noting that the final area is \(A = 0 \text{ m}^2\) and the initial area is \(A_0 = 0.20 \text{ m} \times 0.35 \text{ m}\), and noting that the angle \(\phi\) between the magnetic field and the normal to the surface is 0°, we find that the magnitude of the emf induced in the coil is
\[ |\xi| = -N \frac{BA \cos \phi - BA_0 \cos \phi}{t - t_0} \]
\[ = -\left(1 - \frac{(0.65 \text{ T})(0 \text{ m}^2) \cos 0^\circ - (0.65 \text{ T})(0.20 \text{ m} \times 0.35 \text{ m}) \cos 0^\circ}{0.18 \text{ s}}\right) = 0.25 \text{ V} \]

20. **Reasoning**

An emf is induced in the body because the magnetic flux is changing in time. According to Faraday's law, as given by Equation 22.3, the magnitude of the emf is

\[ |\xi| = -N \frac{\Delta \Phi}{\Delta t} \].

This expression can be used to determine the time interval \( \Delta t \) during which the magnetic field goes from its initial value to zero. The magnetic flux \( \Phi \) is obtained from Equation 22.2 as \( \Phi = BA \cos \phi \), where \( \phi = 0^\circ \) in this problem.

**Solution**

The magnitude \( |\xi| \) of the induced emf is

\[ |\xi| = -N \left( \frac{\Delta \Phi}{\Delta t} \right) = -N \left( \frac{BA \cos \phi - BA_0 \cos \phi}{\Delta t} \right) \]

Solving this relation for \( \Delta t \) gives

\[ \Delta t = \frac{-NA \cos \phi (B - B_0)}{|\xi|} = \frac{-1(0.032 \text{ m}^2) \cos 0^\circ (0 - 1.5 \text{ T})}{0.010 \text{ V}} = 4.8 \text{ s} \]

21. **SSM Reasoning**

According to Equation 22.3, the average emf induced in a coil of \( N \) loops is \( \xi = -N \Delta \Phi / \Delta t \).

**Solution**

For the circular coil in question, the flux through a single turn changes by

\[ \Delta \Phi = BA \cos 45^\circ - BA \cos 90^\circ = BA \cos 45^\circ \]

during the interval of \( \Delta t = 0.010 \text{ s} \). Therefore, for \( N \) turns, Faraday's law gives the magnitude of the emf as

\[ |\xi| = -N \frac{BA \cos 45^\circ}{\Delta t} \]

Since the loops are circular, the area \( A \) of each loop is equal to \( \pi r^2 \). Solving for \( B \), we have

\[ B = \frac{|\xi| \Delta t}{N \pi r^2 \cos 45^\circ} = \frac{(0.065 \text{ V})(0.010 \text{ s})}{(950)\pi(0.060 \text{ m})^2 \cos 45^\circ} = 8.6 \times 10^{-5} \text{ T} \]
22. **REASONING** According to Ohm’s law (see Section 20.2), the resistance of the wire is equal to the emf divided by the current. The emf can be obtained from Faraday’s law of electromagnetic induction.

**SOLUTION** The resistance $R$ of the wire is

$$R = \frac{\xi}{I}$$  \hspace{1cm} (20.2)

According to Faraday’s law of electromagnetic induction, the induced emf is

$$\xi = -N \frac{\Delta \Phi}{\Delta t} = -N \left( \frac{\Phi - \Phi_0}{t - t_0} \right)$$  \hspace{1cm} (22.3)

where $N$ is the number of loops in the coil, $\Phi$ and $\Phi_0$ are, respectively, the final and initial fluxes, and $t - t_0$ is the elapsed time. Substituting Equation 22.3 into Equation 20.2 yields

$$R = \frac{\xi}{I} = \frac{-N}{I} \left( \frac{\Phi - \Phi_0}{t - t_0} \right) = \frac{-12}{230 \text{ A}} \left( \frac{4.0 \text{ Wb} - 9.0 \text{ Wb}}{0.050 \text{ s}} \right) = 5.2 \Omega$$

23. **REASONING** We will use Faraday’s law of electromagnetic induction, Equation 22.3, to find the emf induced in the loop. Once this value has been determined, we can employ Equation 22.3 again to find the rate at which the area changes.

**SOLUTION**

a. The magnitude $|\xi|$ of the induced emf is given by Equation 22.3 as

$$|\xi| = -N \left( \frac{\Phi - \Phi_0}{t - t_0} \right) = -N \left( \frac{BA \cos \phi - B_0 A \cos \phi}{t - t_0} \right) = -NA \cos \phi \left( \frac{B - B_0}{t - t_0} \right)$$

$$= (1)(0.35 \text{ m} \times 0.55 \text{ m}) \cos 65^\circ \left( \frac{2.1 \text{ T} - 0.5 \text{ T}}{0.45 \text{ s} - 0 \text{ s}} \right) = 0.38 \text{ V}$$

b. When the magnetic field is constant and the area is changing in time, Faraday’s law can be written as

$$|\xi| = -N \left( \frac{\Phi - \Phi_0}{t - t_0} \right) = -N \left( \frac{BA \cos \phi - B A_0 \cos \phi}{t - t_0} \right)$$

$$= -NB \cos \phi \left( \frac{A - A_0}{t - t_0} \right) = -NB \cos \phi \left( \frac{\Delta A}{\Delta t} \right)$$
Solving this equation for \( \frac{\Delta A}{\Delta t} \) and substituting in the value of 0.38 V for the magnitude of the emf, we find that

\[
\frac{\Delta A}{\Delta t} = \frac{\xi}{NB \cos \phi} = \frac{0.38 \text{ V}}{(1)(2.1 \text{ T}) \cos 65^\circ} = 0.43 \text{ m}^2/\text{s}
\]

24. **REASONING** According to Faraday's law the emf induced in either single-turn coil is given by Equation 22.3 as

\[
\xi = -N \frac{\Delta \Phi}{\Delta t} = -\frac{\Delta \Phi}{\Delta t}
\]

since the number of turns is \( N = 1 \). The flux is given by Equation 22.2 as

\[
\Phi = BA \cos \phi = BA \cos 0^\circ = BA
\]

where the angle between the field and the normal to the plane of the coil is \( \phi = 0^\circ \), because the field is perpendicular to the plane of the coil and, hence, parallel to the normal. With this expression for the flux, Faraday's law becomes

\[
\xi = -\frac{\Delta \Phi}{\Delta t} = -\frac{\Delta (BA)}{\Delta t} = -A \frac{\Delta B}{\Delta t}
\]

(1)

In this expression we have recognized that the area \( A \) does not change in time and have separated it from the magnitude \( B \) of the magnetic field. We will apply this form of Faraday's law to each coil. The current induced in either coil depends on the resistance \( R \) of the coil, as well as the emf. According to Ohm's law, the current \( I \) induced in either coil is given by

\[
I = \frac{\xi}{R}
\]

(2)

**SOLUTION** Applying Faraday's law in the form of Equation 1 to both coils, we have

\[
\xi_{\text{square}} = -A_{\text{square}} \frac{\Delta B}{\Delta t} = -L^2 \frac{\Delta B}{\Delta t} \quad \text{and} \quad \xi_{\text{circle}} = -A_{\text{circle}} \frac{\Delta B}{\Delta t} = -\pi r^2 \frac{\Delta B}{\Delta t}
\]

The area of a square of side \( L \) is \( L^2 \), and the area of a circle of radius \( r \) is \( \pi r^2 \). The rate of change \( \Delta B/\Delta t \) of the field magnitude is the same for both coils. Dividing these two expressions gives

\[
\frac{\xi_{\text{square}}}{\xi_{\text{circle}}} = \frac{-L^2 \Delta B}{-\pi r^2 \frac{\Delta B}{\Delta t}} = \frac{L^2}{\pi r^2}
\]

(3)

The same wire is used for both coils, so we know that the perimeter of the square equals the circumference of the circle:
\[ 4L = 2\pi r \quad \text{or} \quad \frac{L}{r} = \frac{2\pi}{4} = \frac{\pi}{2} \]

Substituting this result into Equation (3) gives

\[ \frac{\xi_{\text{square}}}{\xi_{\text{circle}}} = \frac{I^2}{\pi r^2} = \frac{\pi^2}{\pi 2^2} = \frac{\pi}{4} \quad \text{or} \quad \frac{\xi_{\text{square}}}{\xi_{\text{circle}}} = \frac{\pi}{4} \left( \frac{0.80 \text{ V}}{\xi_{\text{circle}}} \right) = 0.63 \text{ V} \]  

(4)

Using Ohm’s law as given in Equation (2), we find for the induced currents that

\[ \frac{I_{\text{square}}}{I_{\text{circle}}} = \frac{\xi_{\text{square}}}{\xi_{\text{circle}}} = \frac{\pi}{4} \]

Here we have used the fact that the same wire has been used for each coil, so that the resistance \( R \) is the same for each coil. Using the result in Equation (4) gives

\[ \frac{I_{\text{square}}}{I_{\text{circle}}} = \frac{\pi}{4} \left( \frac{3.2 \text{ A}}{\xi_{\text{circle}}} \right) = 2.5 \text{ A} \]

25. REASONING The magnitude \( |\xi| \) of the average emf induced in the triangle is given by

\[ |\xi| = -N \frac{\Delta \Phi}{\Delta t} \]  

(see Equation 22.3), which is Faraday’s law. This expression can be used directly to calculate the magnitude of the average emf. Since the triangle is a single-turn coil, the number of turns is \( N = 1 \). According to Equation 22.2, the magnetic flux \( \Phi \) is

\[ \Phi = BA \cos \phi = BA \cos 0^\circ = BA \]  

(1)

where \( B \) is the magnitude of the field, \( A \) is the area of the triangle, and \( \phi = 0^\circ \) is the angle between the field and the normal to the plane of the triangle (the magnetic field is perpendicular to the plane of the triangle). It is the change \( \Delta \Phi \) in the flux that appears in Faraday’s law, so that we use Equation (1) as follows:

\[ \Delta \Phi = BA - BA_0 = BA \]

where \( A_0 = 0 \text{ m}^2 \) is the initial area of the triangle just as the bar passes point A, and \( A \) is the area after the time interval \( \Delta t \) has elapsed. The area of a triangle is one-half the base \( (d_{AC}) \) times the height \( (d_{CB}) \) of the triangle. Thus, the change in flux is

\[ \Delta \Phi = BA = B \left( \frac{1}{2} d_{AC} d_{CB} \right) \]

The base and the height of the triangle are related, according to \( d_{CB} = d_{AC} \tan \theta \), where \( \theta = 19^\circ \). Furthermore, the base of the triangle becomes longer as the rod moves. Since the rod moves with a speed \( v \) during the time interval \( \Delta t \), the base is \( d_{AC} = v \Delta t \). With these substitutions the change in flux becomes
\[
\Delta \Phi = B \left( \frac{1}{2} d_{AC} d_{CB} \right) = B \left[ \frac{1}{2} d_{AC} \left( d_{AC} \tan \theta \right) \right] = \frac{1}{2} B (v \Delta t)^2 \tan \theta
\] (2)

**SOLUTION** Substituting Equation (2) for the change in flux into Faraday’s law, we find that the magnitude of the induced emf is

\[
|\xi| = \left| -N \frac{\Delta \Phi}{\Delta t} \right| = \left| -N \frac{1}{2} B (v \Delta t)^2 \tan \theta \right| = N \frac{1}{2} B v^2 \Delta t \tan \theta
\]

\[
= (1 \frac{1}{2}) (0.38 \text{ T}) (0.60 \text{ m/s})^2 (6.0 \text{ s}) \tan 19^\circ = 0.14 \text{ V}
\]

26. **REASONING** An emf is induced in the coil because the magnetic flux through the coil is changing in time. The flux is changing because the angle \(\phi\) between the normal to the coil and the magnetic field is changing.

The amount of induced current is equal to the induced emf divided by the resistance of the coil (see Equation 20.2).

According to Equation 20.1, the amount of charge \(\Delta q\) that flows is \(\Delta q = I \Delta t\) during which the coil rotates, or \(\Delta q = I(t - t_0)\).

**SOLUTION** According to Equation 20.1, the amount of charge that flows is \(\Delta q = I \Delta t\). The current is related to the emf \(\xi\) in the coil and the resistance \(R\) by Equation 20.2 as \(I = \xi / R\). The amount of charge that flows can, therefore, be written as

\[
\Delta q = I \Delta t = \left( \frac{\xi}{R} \right) \Delta t
\]

The emf is given by Faraday’s law of electromagnetic induction as

\[
\xi = -N \left( \frac{\Delta \Phi}{\Delta t} \right) = -N \left( \frac{BA \cos \phi - BA \cos \phi_0}{\Delta t} \right)
\]

where we have also used Equation 22.2, which gives the definition of magnetic flux as \(\Phi = BA \cos \phi\). With this emf, the expression for the amount of charge becomes

\[
\Delta q = I \Delta t = \left[ -N \left( \frac{BA \cos \phi - BA \cos \phi_0}{\Delta t} \right) \right] \Delta t = \frac{-NBA(\cos \phi - \cos \phi_0)}{R}
\]

Solving for the magnitude of the magnetic field yields

\[
B = \frac{-R \Delta q}{NA(\cos \phi - \cos \phi_0)} = \frac{- (140 \Omega)(8.5 \times 10^{-3} \text{ C})}{(50)(1.5 \times 10^{-3} \text{ m}^2)(\cos 90^\circ - \cos 0^\circ)} = 0.16 \text{ T}
\]
27. **Reasoning** According to Equation 22.3, the average emf $\xi$ induced in a single loop ($N = 1$) is $\xi = -\Delta \Phi / \Delta t$. Since the magnitude of the magnetic field is changing, the area of the loop remains constant, and the direction of the field is parallel to the normal to the loop, the change in flux through the loop is given by $\Delta \Phi = (\Delta B) A$. Thus the magnitude $|\xi|$ of the induced emf in the loop is given by $|\xi| = -(\Delta B) A / \Delta t$.

Similarly, when the area of the loop is changed and the field $B$ has a given value, we find the magnitude of the induced emf to be $|\xi| = -B(\Delta A) / \Delta t$.

**Solution**

a. The magnitude of the induced emf when the field changes in magnitude is

$$|\xi| = -A \left( \frac{\Delta B}{\Delta t} \right) = (0.018 \text{ m}^2)(0.20 \text{ T/s}) = 3.6 \times 10^{-3} \text{ V}$$

b. At a particular value of $B$ (when $B$ is changing), the rate at which the area must change can be obtained from

$$|\xi| = \left| \frac{B \Delta A}{\Delta t} \right| \text{ or } \frac{\Delta A}{\Delta t} = \frac{|\xi|}{B} = \frac{3.6 \times 10^{-3} \text{ V}}{1.8 \text{ T}} = 2.0 \times 10^{-3} \text{ m}^2 / \text{s}$$

In order for the induced emf to be zero, the magnitude of the magnetic field and the area of the loop must change in such a way that the flux remains constant. Since the magnitude of the magnetic field is increasing, the area of the loop must decrease, if the flux is to remain constant. Therefore, **the area of the loop must be shrunk**.

28. **Reasoning** The magnitude $B_1$ of the magnetic field at the center of a circular coil with $N$ turns and a radius $r$, carrying a current $I$ is given by

$$B_1 = N \frac{\mu_0 I}{2r} \quad (21.6)$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space. We use the symbol $r$ to denote the radius of the coil in order to distinguish it from the coil’s resistance $R$. The coil’s induced current $I$ is caused by the induced emf $\xi$. We will determine the induced current with Ohm’s law (Equation 20.2), the magnitude $|\xi|$ of the induced emf, and the resistance $R$ of the coil:

$$I = \frac{|\xi|}{R} \quad (20.2)$$

The magnitude of the induced emf is found from Faraday’s law: $|\xi| = -N \frac{\Delta \Phi}{\Delta t}$ (Equation 22.3), where $\Delta \Phi / \Delta t$ is the rate of change of the flux caused by the external
magnetic field $B$. At any instant, the flux through each turn of the coil due to the external magnetic field is given by

$$\Phi = BA \cos \phi = BA \cos 0^\circ = BA$$ (22.2)

where $A$ is the cross-sectional area of the coil. The angle $\phi$ between the direction of the external magnetic field and the normal to the plane of the coil is zero, because the external magnetic field is perpendicular to the plane of the coil.

**SOLUTION** In this situation, the cross-sectional area $A$ of the coil is constant, but the magnitude $B$ of the external magnetic field is changing. Therefore, substituting Equation 22.2 into $|\xi| = -N \frac{\Delta \Phi}{\Delta t}$ (Equation 22.3) yields

$$|\xi| = -N \frac{\Delta \Phi}{\Delta t} = N \frac{\Delta (BA)}{\Delta t} = NA \frac{\Delta B}{\Delta t}$$ (1)

Substituting Equation (1) into Equation 20.2, we obtain

$$I = \frac{|\xi|}{R} = \frac{NA \frac{\Delta B}{\Delta t}}{R} = NA \left( \frac{\Delta B}{\Delta t} \right)$$ (2)

Substituting Equation (2) into Equation (21.6) yields

$$B_1 = N \frac{\mu_0 I}{2r} = N \frac{\mu_0}{2r} \left( \frac{NA}{R} \right) \left( \frac{\Delta B}{\Delta t} \right) = N^2 \frac{\mu_0 A}{2rR} \left( \frac{\Delta B}{\Delta t} \right)$$ (3)

Lastly, we substitute $A = \pi r^2$ into Equation (3), because the coil is circular. This yields

$$B_1 = N^2 \frac{\mu_0 \pi r^2}{2r \frac{R}{2}} \left( \frac{\Delta B}{\Delta t} \right) = N^2 \frac{\mu_0 \pi r}{2} \left( \frac{\Delta B}{\Delta t} \right)$$

$$= (105)^2 \frac{(4\pi \times 10^{-7} \ T \cdot m/A) \pi (4.00 \times 10^{-2} \ m)}{2 (0.480 \ \Omega)} (0.783 \ T/s) = 1.42 \times 10^{-3} \ T$$ (4)

**29. REASONING** A greater magnetic flux passes through the coil in part $b$ of the drawing. The flux is given as $\Phi = BA \cos \phi$ (Equation 22.2), where $B$ is the magnitude of the magnetic field, $A$ is the area of the surface through which the field passes, and $\phi$ is the angle between the normal to the plane of the surface and the field. In part $b$ the coil area has two parts, and so does the flux. There is the larger semicircular area $\left( \frac{1}{2} \pi r^2_{\text{larger}} \right)$ on the horizontal surface and the smaller semicircular $\left( \frac{1}{2} \pi r^2_{\text{smaller}} \right)$ area perpendicular to the horizontal surface. The field is parallel to the normal for the larger area and perpendicular to the normal for the smaller area.
Hence, the flux is

\[
\Phi_b = BA_{\text{lager}} \cos \phi_{\text{lager}} + BA_{\text{smaller}} \cos \phi_{\text{smaller}}
\]

\[
= B \left( \frac{1}{2} \pi r_{\text{lager}}^2 \right) \cos 0^\circ + B \left( \frac{1}{2} \pi r_{\text{smaller}}^2 \right) \cos 90^\circ = B \left( \frac{1}{2} \pi r_{\text{lager}}^2 \right)
\]  

(1)

In part \(a\) the entire area of the coil lies on the horizontal surface and is the area between the two semicircles. The plane of the coil is perpendicular to the magnetic field. The flux, therefore, is

\[
\Phi_a = BA \cos \phi = B \left( \frac{1}{2} \pi r_{\text{lager}}^2 - \frac{1}{2} \pi r_{\text{smaller}}^2 \right) \cos 0^\circ = B \left( \frac{1}{2} \pi r_{\text{lager}}^2 - \frac{1}{2} \pi r_{\text{smaller}}^2 \right)
\]  

(2)

which is smaller than the flux in part \(b\) of the drawing.

According to Lenz’s law the induced magnetic field associated with the induced current opposes the change in flux. To oppose the increase in flux from part \(a\) to part \(b\) of the drawing, the induced field must point downward. In this way, it will reduce the effect of the increasing coil area available to the upward-pointing external field. An application of RHR-2 shows that the induced current must flow clockwise in the larger semicircle of wire (when viewed from above) in order to create a downward-pointing induced field. The average induced current can be determined according to Ohm’s law as the average induced emf divided by the resistance of the coil.

The period \(T\) of the rotational motion is related to the angular frequency \(\omega\) according to

\[\omega = \frac{2\pi}{T}\] (Equation 10.6). The shortest time interval \(\Delta t\) that elapses between parts \(a\) and \(b\) of the drawing is the time needed for one quarter of a turn or \(\frac{1}{4}T\).

**SOLUTION** According to Ohm’s law, the average induced current is

\[I = \frac{\xi}{R}\]

where \(\xi\) is the average induced emf and \(R\) is the resistance of the coil. According to Faraday’s law, Equation 22.3, the average induced emf is \[\xi = -\frac{\Delta \Phi}{\Delta t}\]

where we have set \(N = 1\). We can determine the change \(\Delta \Phi\) in the flux as \(\Delta \Phi = \Phi_b - \Phi_a\), where the fluxes in
parts $a$ and $b$ of the drawing are available from Equations (1) and (2). The time interval $\Delta t$ is $\frac{1}{4}T$, as discussed in the **REASONING**. Using Equation 10.6, we have for the period that $T = \frac{2\pi}{\omega}$. Thus, we find for the current that

$$I = \frac{\xi}{R} = \frac{-\Delta \Phi}{\Delta t} = \frac{-\left(\Phi_b - \Phi_a\right)}{\frac{1}{4}T}$$

$$= \frac{B\left(\frac{1}{2}\pi r_{\text{larger}}^2\right) - B\left(\frac{1}{2}\pi r_{\text{larger}}^2 - \frac{1}{2}\pi r_{\text{smaller}}^2\right)}{R\left(\frac{1}{4}T\right)} = \frac{B\left(\frac{1}{2}\pi r_{\text{smaller}}^2\right)}{R\left(\frac{1}{4}T\right)} = \frac{B\left(\frac{1}{2}\pi r_{\text{smaller}}^2\right)}{R\left(\frac{1}{4}(2\pi/\omega)\right)}$$

$$= \frac{-0.35 \text{ T}\left(\frac{1}{2}\pi (0.20 \text{ m})^2\right)}{(0.025 \Omega)\left(\frac{1}{4}\pi (1.5 \text{ rad/s})\right)} = -0.84 \text{ A}$$

---

30. **REASONING**  We can obtain the magnitude $B$ of the magnetic field with the aid of Faraday's law, which is $\xi = -N \frac{\Delta \Phi}{\Delta t}$ (Equation 22.3), where $\xi$ is the emf induced in the coil, $N = 1850$ is the number of turns in the coil, $\Delta \Phi$ is the change in magnetic flux, and $\Delta t$ is the time interval during which the flux changes. The flux is $\Phi = BA\cos \phi$ (Equation 22.3), where $A = 4.70 \times 10^{-4} \text{ m}^2$ is the area of each turn of the coil and $\phi = 0^\circ$ is the angle between the normal to the coil and the field. The emf can be related to the induced charge that flows in the coil by using Ohm's law $\xi = IR$ (Equation 20.2), where $R = 45.0 \Omega$ is the total resistance of the circuit and $I$ is the current. The current is given by $I = \frac{\Delta q}{\Delta t}$ (Equation 20.1), where $\Delta q = 8.87 \times 10^{-3} \text{ C}$ is the amount of charge that flows in the time interval $\Delta t$. We have no value for $\Delta t$. However, it will be eliminated algebraically from our calculation, as we will see.

**SOLUTION** Using Equation 22.3, we can write the change in flux as

$$\Delta \Phi = BA\cos \phi - 0 = BA\cos 0^\circ = BA$$

With this change in flux, Faraday's law (without the minus sign, since we seek only the magnitude of the field) becomes

$$\xi = N \frac{\Delta \Phi}{\Delta t} = N \frac{BA}{\Delta t} \quad \text{or} \quad B = \frac{\xi (\Delta t)}{NA}$$
Combining Ohm’s law and Equation 20.1 for the current, we obtain \( \xi = IR = \left( \frac{\Delta q}{\Delta t} \right) R \).

Substituting this result for \( \xi \) into the expression for \( B \), we obtain

\[
B = \frac{\xi (\Delta t)}{NA} = \left( \frac{\Delta q}{\Delta t} \right) \frac{R (\Delta t)}{NA} = \left( \frac{\Delta q}{\Delta t} \right) \frac{R}{NA} = \frac{8.87 \times 10^{-3} \text{ C}}{1850 (4.70 \times 10^{-4} \text{ m}^2)} = 0.459 \text{T}
\]

31. **REASONING AND SOLUTION** Consider one revolution of either rod. The magnitude \( |\xi| \) of the emf induced across the rod is

\[
|\xi| = -B \frac{\Delta A}{\Delta t} = \frac{B (\pi L^2)}{\Delta t}
\]

The angular speed of the rods is \( \omega = \frac{2\pi}{\Delta t} \), so \( |\xi| = \frac{1}{2} BL^2 \omega \). The rod tips have opposite polarity since they are rotating in opposite directions. Hence, the difference in potentials of the tips is

\[
\Delta V = BL^2 \omega
\]

so

\[
\omega = \frac{\Delta V}{BL^2} = \frac{4.5 \times 10^3 \text{ V}}{4.7 \text{ T} (0.68 \text{ m})^2} = 2100 \text{ rad/s}
\]

32. **REASONING** Our solution is based on Lenz’s law. According to Lenz’s law, the induced emf has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change. We will also have need of right-hand rule no. 2 (RHR-2) as we determine which end of the resistor is the positive end.

**SOLUTION** As the loop rotates through one-half a revolution, the area through which the magnetic field passes decreases. The magnetic flux, being proportional to the area, also decreases. The induced magnetic field must oppose this decrease in flux, so the induced magnetic field must strengthen the original magnetic field. Thus, the induced magnetic field points in the same direction as the original magnetic field, or into the plane of the page. According to RHR-2, the induced current flows clockwise around the loop (and right-to-left through the resistor) so that its magnetic field will be directed into the page. Since conventional current flows through a resistor from higher potential toward lower potential, [the right end of the resistor must be the positive end].
33. **REASONING** The external magnetic field is perpendicular to the plane of the horizontal loop, so it must point either upward or downward. We will use Lenz’s law to decide whether the external magnetic field $B$ field points up or down. This law predicts that the direction of the induced magnetic field $B_{\text{ind}}$ opposes the change in the magnetic flux through the loop due to the external field.

**SOLUTION** The external magnetic field $B$ is increasing in magnitude, so that the magnetic flux through the loop also increases with time. In order to oppose the increase in magnetic flux, the induced magnetic field $B_{\text{ind}}$ must be directed opposite to the external magnetic field $B$. The drawing shows the loop as viewed from above, with an induced current $I_{\text{ind}}$ flowing clockwise. According to Right-Hand Rule No. 2 (see Section 21.7), this induced current creates an induced magnetic field $B_{\text{ind}}$ that is directed into the page at the center of the loop (and all other points of the loop’s interior). Therefore, the external magnetic field $B$ must be directed out of the page. Because we are viewing the loop from above, “out of the page” corresponds to upward toward the viewer.

34. **REASONING** The magnetic field produced by $I$ extends throughout the space surrounding the loop. Using RHR-2, it can be shown that the magnetic field is parallel to the normal to the loop. Thus, the magnetic field penetrates the loop and generates a magnetic flux.

According to Faraday’s law of electromagnetic induction, an emf is induced when the magnetic flux through the loop is changing in time. If the current $I$ is constant, the magnetic flux is constant, and no emf is induced in the loop. However, if the current is decreasing in time, the magnetic flux is decreasing and an induced current exists in the loop.

Lenz’s law states that the induced magnetic field opposes the change in the magnetic field produced by the current $I$. The induced magnetic field does not necessarily oppose the magnetic field itself. Thus, the induced magnetic field does not always have a direction that is opposite to the direction of the field produced by $I$.

**SOLUTION** At the location of the loop, the magnetic field produced by the current $I$ is directed into the page (this can be verified by using RHR-2). The current is decreasing, so the magnetic field is decreasing. Therefore, the magnetic flux that penetrates the loop is decreasing. According to Lenz’s law, the induced emf has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes this flux change. The induced magnetic field will oppose this decrease in flux by pointing into the page, in the same direction as the field produced by $I$. According to RHR-2, the induced current must flow clockwise around the loop in order to produce such an induced field. The current then flows from left-to-right through the resistor.
35. **REASONING** The current $I$ produces a magnetic field, and hence a magnetic flux, that passes through the loops A and B. Since the current decreases to zero when the switch is opened, the magnetic flux also decreases to zero. According to Lenz's law, the current induced in each coil will have a direction such that the induced magnetic field will oppose the original flux change.

**SOLUTION**

a. The drawing in the text shows that the magnetic field at coil A is perpendicular to the plane of the coil and points down (when viewed from above the table top). When the switch is opened, the magnetic flux through coil A decreases to zero. According to Lenz's law, the induced magnetic field produced by coil A must oppose this change in flux. Since the magnetic field points down and is decreasing, the induced magnetic field must also point down. According to Right-Hand Rule No. 2 (RHR-2), the induced current must be **clockwise** around loop A.

b. The drawing in the text shows that the magnetic field at coil B is perpendicular to the plane of the coil and points up (when viewed from above the table top). When the switch is opened, the magnetic flux through coil B decreases to zero. According to Lenz's law, the induced magnetic field produced by coil B must oppose this change in flux. Since the magnetic field points up and is decreasing, the induced magnetic field must also point up. According to RHR-2, the induced current must be **counterclockwise** around loop B.

36. **REASONING** According to Lenz's law, the induced current in the triangular loop flows in such a direction so as to create an induced magnetic field that opposes the original flux change.

**SOLUTION**

a. As the triangle is crossing the $+y$ axis, the magnetic flux down into the plane of the paper is increasing, since the field now begins to penetrate the loop. To offset this increase, an induced magnetic field directed up and out of the plane of the paper is needed. By applying RHR-2 it can be seen that such an induced magnetic field will be created within the loop by a **counterclockwise induced current**.

b. As the triangle is crossing the $-x$ axis, there is no flux change, since all parts of the triangle remain in the magnetic field, which remains constant. Therefore, there is no induced magnetic field, and **no induced current appears**.

c. As the triangle is crossing the $-y$ axis, the magnetic flux down into the plane of the paper is decreasing, since the loop now begins to leave the field region. To offset this decrease, an induced magnetic field directed down and into the plane of the paper is needed. By applying RHR-2 it can be seen that such an induced magnetic field will be created within the loop by a **clockwise induced current**.

d. As the triangle is crossing the $+x$ axis, there is no flux change, since all parts of the triangle remain in the field-free region. Therefore, there is no induced magnetic field, and **no induced current appears**.
37. **REASONING** The current \( I \) in the straight wire produces a circular pattern of magnetic field lines around the wire. The magnetic field at any point is tangent to one of these circular field lines. Thus, the field points perpendicular to the plane of the table. Furthermore, according to Right-Hand Rule No. 2, the field is directed up out of the table surface in region 1 above the wire and is directed down into the table surface in region 2 below the wire (see the drawing at the right). To deduce the direction of any induced current in the circular loop, we consider Faraday’s law and the change that occurs in the magnetic flux through the loop due to the field of the straight wire.

**SOLUTION** As the current \( I \) decreases, the magnitude of the field that it produces also decreases. However, the directions of the fields in regions 1 and 2 do not change and remain as discussed in the reasoning. Since the fields in these two regions always have opposite directions and equal magnitudes at any given radial distance from the straight wire, the flux through the regions add up to give zero for any value of the current. With the flux remaining constant as time passes, Faraday’s law indicates that there is no induced emf in the coil. Since there is no induced emf in the coil, there is no induced current.

38. **REASONING** Our solution is based on Lenz’s law. According to Lenz’s law, the induced emf has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change. As we refer to the drawing at the right, we will also have need of right-hand rule no. 2 (RHR-2). Note that the horizontal arrows in the drawing indicate induced current that exists in the ring.

**SOLUTION**

a. When the magnet is above the ring its magnetic field points down through the ring and is increasing in strength as the magnet falls. According to Lenz’s law, an induced magnetic field appears that attempts to reduce the increasing field. Therefore, the induced field must point up. Using RHR-2, we can see that the induced current in the ring is as shown in the drawing. Because of the induced current, the ring looks like a magnet with its north pole at the top (use RHR-2). The north pole of the loop repels the falling magnet and retards its motion. When the magnet is below the ring, its magnetic field still points down through the ring but is decreasing as the magnet falls. According to Lenz’s law, the induced magnetic field attempts to bolster the decreasing field and, therefore, must point down. The induced current in the ring is as shown in the drawing, and the ring then looks like a magnet with its
north pole at the bottom, attracting the south pole of the falling magnet and retarding its motion.

b. The motion of the magnet is unaffected, since no induced current can flow in the cut ring. No induced current means that no induced magnetic field can be produced to repel or attract the falling magnet.

39. **REASONING AND SOLUTION**
   a. **Location I**
   As the loop swings downward, the normal to the loop makes a smaller angle with the applied field. Hence, the flux through the loop is increasing. The induced magnetic field must point generally to the left to counteract this increase. The induced current flows

   $$x \rightarrow y \rightarrow z$$

   **Location II**
   The angle between the normal to the loop and the applied field is now increasing, so the flux through the loop is decreasing. The induced field must now be generally to the right, and the current flows

   $$z \rightarrow y \rightarrow x$$

   b. **Location I**
   The argument is the same as for location II in part a.

   $$z \rightarrow y \rightarrow x$$

   **Location II**
   The argument is the same as for location I in part a.

   $$x \rightarrow y \rightarrow z$$

40. **REASONING** When the motor is running at normal speed, the current is the net emf divided by the resistance $R$ of the armature wire. The net emf is the applied voltage $V$ minus the back emf developed by the rotating coil. We can use this relation to find the back emf. When the motor is just turned on, there is no back emf, so the current is just the applied voltage divided by the resistance of the wire. To limit the starting current to 15.0 A, a resistor $R_1$ is placed in series with the resistance of the wire, so the equivalent resistance is $R + R_1$. The current to the motor is equal to the applied voltage divided by the equivalent resistance.

   **SOLUTION**
   a. According to Equation 22.5, the back emf $\xi$ generated by the motor is

   $$\xi = V - IR = 120.0 \text{ V} - (7.00 \text{ A})(0.720 \text{ } \Omega) = 115 \text{ V}$$

   b. When the motor has been just turned on, the back emf is zero, so the current is
\[ I = \frac{V - \xi}{R} = \frac{120.0 \, \text{V} - 0 \, \text{V}}{0.720 \, \Omega} = 167 \, \text{A} \]

c. When a resistance \( R_1 \) is placed in series with the resistance \( R \) of the wire, the equivalent resistance is \( R_1 + R \). The current to the motor is

\[ I = \frac{V - \xi}{R_1 + R} \]

Solving this expression for \( R_1 \) gives

\[ R_1 = \frac{V - \xi}{I} - R = \frac{120.0 \, \text{V} - 0 \, \text{V}}{15.0 \, \text{A}} - 0.720 \, \Omega = 7.28 \, \Omega \]

41. **Reasoning** We can use the information given in the problem statement to determine the area of the coil \( A \). Since it is square, the length of one side is \( \ell = \sqrt{A} \).

**Solution** According to Equation 22.4, the maximum emf \( \xi_0 \) induced in the coil is \( \xi_0 = NAB\omega \). Therefore, the length of one side of the coil is

\[ \ell = \sqrt{A} = \frac{\xi_0}{NB\omega} = \sqrt{\frac{75.0 \, \text{V}}{(248)(0.170 \, \text{T})(79.1 \, \text{rad/s})}} = 0.150 \, \text{m} \]

42. **Reasoning** The emf \( \xi \) of the generator is \( \xi = NAB\omega \sin \omega t \) (Equation 22.4), where \( N = 150 \) is the number of turns in the coil, \( A = 0.85 \, \text{m}^2 \) is the area per turn, \( B \) is the magnitude of the magnetic field, \( \omega \) is the angular speed in rad/s, and \( t \) is the time. The angular speed is \( \omega = 2\pi f \) (Equation 10.6), where \( f = 60.0 \, \text{Hz} \) is the frequency in cycles per second or Hertz. Equation 22.4 indicates that the maximum emf is \( \xi_{\max} = NAB\omega \), since \( \sin \omega t \) has a maximum value of 1.

**Solution** Substituting \( \omega = 2\pi f \) into \( \xi_{\max} = NAB\omega \) and solving for the result for \( B \) reveals that

\[ B = \frac{\xi_{\max}}{NA(2\pi f)} = \frac{5500 \, \text{V}}{150(0.85 \, \text{m}^2)(2\pi(60.0 \, \text{Hz}))} = 0.11 \, \text{T} \]

43. **Reasoning** The number \( N \) of turns in the coil of a generator is given by \( N = \frac{\xi_0}{AB\omega} \) (Equation 22.4), where \( \xi_0 \) is the peak emf, \( A \) is the area per turn, \( B \) is the magnitude of the magnetic field, and \( \omega \) is the angular speed in rad/s. We have values for \( A \) and \( B \). Although we are not given the peak emf \( \xi_0 \), we know that it is related to the rms emf, which is known:
\( \xi_0 = \sqrt{2} \xi_{\text{rms}} \) (Equation 20.13). We are also not given the angular speed \( \omega \), but we know that it is related to the frequency \( f \) in hertz according to \( \omega = 2\pi f \) (Equation 10.6).

**SOLUTION** Substituting Equation 20.13 for \( \xi_0 \) and Equation 10.6 for \( \omega \) into Equation 22.4 gives

\[
N = \frac{\xi_0}{AB\omega} = \frac{\sqrt{2} \xi_{\text{rms}}}{AB2\pi f} = \frac{\sqrt{2} (120 \text{ V})}{(0.022 \text{ m}^2)(6.9 \times 10^{-5} \text{ T})(2\pi(60.0 \text{ Hz}))} = 3.0 \times 10^5
\]

44. **REASONING AND SOLUTION** Using Equation 22.5 to take the back emf into account, we find

\[
R = \frac{V - \xi}{I} = \frac{(120.0 \text{ V}) - (72.0 \text{ V})}{3.0 \text{ A}} = 16 \Omega
\]

45. **REASONING** The length of the wire is the number \( N \) of turns times the length per turn. Since each turn is a square that is a length \( L \) on a side, the length per turn is \( 4L \), and the total length is \( N(4L) \). Since the area of a square is \( A = L^2 \), the length of a side of the square can be obtained as \( L = \sqrt{A} \), so that the total length is \( N(4\sqrt{A}) \). The area can be found from the peak emf, which is \( \xi_0 = NAB\omega \), according to Equation 22.4. Solving this expression for \( A \) and substituting the result into the expression for the total length gives

\[
\text{Total length} = N(4\sqrt{A}) = N\left(4\sqrt{\frac{\xi_0}{N\omega}}\right) = 4\sqrt{\frac{N\xi_0}{B\omega}}
\]

Although we are not given the peak emf \( \xi_0 \), we know that it is related to the rms emf, which is known: \( \xi_0 = \sqrt{2} \xi_{\text{rms}} \) (Equation 20.13). We are also not given the angular speed \( \omega \), but we know that it is related to the frequency \( f \) in hertz according to \( \omega = 2\pi f \) (Equation 10.6).

**SOLUTION** According to Equation 20.13, the peak emf is

\[
\xi_0 = \sqrt{2} \xi_{\text{rms}} = \sqrt{2} (120 \text{ V}) = 170 \text{ V}
\]

Substituting \( \omega = 2\pi f \) and the value for the peak emf into Equation (1) gives

\[
\text{Total length} = 4\sqrt{\frac{N\xi_0}{B\omega}} = 4\sqrt{\frac{N\xi_0}{B2\pi f}} = 4\sqrt{\frac{100(170 \text{ V})}{(0.50 \text{ T})(2\pi(60.0 \text{ Hz}))}} = 38 \text{ m}
\]
46. **REASONING** The emf $\xi$ of the generator is $\xi = NAB\omega \sin \omega t$ (Equation 22.4), where $N$ is the number of turns in the coil, $A$ is the area per turn, $B = 0.20 \text{ T}$ is the magnitude of the magnetic field, $\omega = 25 \text{ rad/s}$ is the angular speed of the coil, and $t$ is the time. Equation 22.4 indicates that the peak or maximum emf is $\xi_{\text{peak}} = NAB\omega$, since $\sin \omega t$ has a maximum value of 1. Recognizing that the area of a circle can be calculated from its radius, we can apply this equation directly to determine $\xi_{\text{peak}}$. However, we do not have a value for the number of turns $N$. To determine it, we will utilize the length $L = 5.7 \text{ m}$ of wire in the coil and the coil's circumference, which can be calculated from radius $R = 0.14 \text{ m}$.

**SOLUTION** Using $A = \pi R^2$ for the area of a circle, we have

$$\xi_{\text{peak}} = NAB\omega = N\left(\pi R^2\right)B\omega$$

The number of turns is the length $L$ of the wire divided by the circumference $2\pi R$ of the circular coil or $N = L/(2\pi R)$. Substituting this expression for $N$ into the equation for $\xi_{\text{peak}}$, we obtain

$$\xi_{\text{peak}} = \left(\frac{L}{2\pi R}\right)\left(\pi R^2\right)B\omega$$

$$= \frac{LRB\omega}{2} = \frac{(5.7 \text{ m})(0.14 \text{ m})(0.20 \text{ T})(25 \text{ rad/s})}{2} = 2.0 \text{ V}$$

47. **SSM** **REASONING** The peak emf $\xi_0$ produced by a generator is related to the number $N$ of turns in the coil, the area $A$ of the coil, the magnitude $B$ of the magnetic field, and the angular speed $\omega_{\text{coil}}$ of the coil by $\xi_0 = NAB\omega_{\text{coil}}$ (see Equation 22.4). We are given that the angular speed $\omega_{\text{coil}}$ of the coil is 38 times as great as the angular speed $\omega_{\text{tire}}$ of the tire. Since the tires roll without slipping, the angular speed of a tire is related to the linear speed $v$ of the bike by $\omega_{\text{tire}} = v/r$ (see Section 8.6), where $r$ is the radius of a tire. The speed of the bike after 5.1 s can be found from its acceleration and the fact that the bike starts from rest.

**SOLUTION** The peak emf produced by the generator is

$$\xi_0 = NAB\omega_{\text{coil}}$$

(22.4)

Since the angular speed of the coil is 38 times as great as the angular speed of the tire, $\omega_{\text{coil}} = 38\omega_{\text{tire}}$. Substituting this expression into Equation 22.4 gives $\xi_0 = NAB\omega_{\text{coil}} = NAB(38\omega_{\text{tire}})$. Since the tire rolls without slipping, the angular speed of the tire is related to the linear speed $v$ (the speed at which its axle is moving forward) by $\omega_{\text{tire}} = v/r$ (Equation 8.12), where $r$ is the radius of the tire. Substituting this result into the expression for $\xi_0$ yields
\[ \xi_0 = NAB\left(38\omega_{tire}\right) = NAB\left(38\frac{v}{r}\right) \]  

(1)

The velocity of the car is given by \( v = v_0 + at \) (Equation 2.4), where \( v_0 \) is the initial velocity, \( a \) is the acceleration and \( t \) is the time. Substituting this relation into Equation (1), and noting that \( v_0 = 0 \text{ m/s} \) since the bike starts from rest, we find that

\[
\xi_0 = NAB\left(38\frac{v}{r}\right) = NAB\left[38\left(\frac{v_0 + at}{r}\right)\right] = (125)(3.86 \times 10^{-3} \text{ m}^2)(0.0900 \text{ T})(38)\left[\frac{0 \text{ m/s} + (0.550 \text{ m/s}^2)(5.10 \text{ s})}{0.300 \text{ m}}\right] = 15.4 \text{ V}
\]

48. **REASONING AND SOLUTION**
   a. On startup, the back emf of the generator is zero. Then,
   \[ R = \frac{V}{I} = \frac{117 \text{ V}}{12.2 \text{ A}} = 9.59 \text{ \Omega} \]
   b. At normal speed
   \[ \xi = V - IR = 117 \text{ V} - (2.30 \text{ A})(9.59 \text{ \Omega}) = 95 \text{ V} \]
   c. The back emf of the motor is proportional to the rotational speed, so at \( \frac{1}{3} \) the normal speed, the back emf is
   \[ \frac{1}{3} (95 \text{ V}) = 32 \text{ V} \]

   The voltage applied to the resistor is then \( V = 117 \text{ V} - 32 \text{ V} = 85 \text{ V} \), so the current is
   \[ I = \frac{V}{R} = \frac{85 \text{ V}}{9.59 \text{ \Omega}} = 8.9 \text{ A} \]

49. **SSM REASONING** The energy density is given by Equation 22.11 as

\[ \text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2\mu_0} B^2 \]

The energy stored is the energy density times the volume.

**SOLUTION** The volume is the area \( A \) times the height \( h \). Therefore, the energy stored is

\[ \text{Energy} = \frac{B^2 A h}{2\mu_0} = \frac{(7.0 \times 10^{-5} \text{ T})^2(5.0 \times 10^8 \text{ m}^2)(1500 \text{ m})}{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})} = 1.5 \times 10^9 \text{ J} \]
50. **REASONING** When the current through an inductor changes, the induced emf $\xi$ is given by Equation 22.9 as

$$\xi = -L \frac{\Delta I}{\Delta t}$$

where $L$ is the inductance, $\Delta I$ is the change in the current, and $\Delta t$ is the time interval during which the current changes. For each interval, we can determine $\Delta I$ and $\Delta t$ from the graph.

**SOLUTION**

a. $$\xi = -L \frac{\Delta I}{\Delta t} = -\left(3.2 \times 10^{-3} \text{ H}\right) \left(\frac{4.0 \text{ A} - 0 \text{ A}}{2.0 \times 10^{-3} \text{ s} - 0 \text{ s}}\right) = -6.4 \text{ V}$$

b. $$\xi = -L \frac{\Delta I}{\Delta t} = -\left(3.2 \times 10^{-3} \text{ H}\right) \left(\frac{4.0 \text{ A} - 4.0 \text{ A}}{5.0 \times 10^{-3} \text{ s} - 2.0 \times 10^{-3} \text{ s}}\right) = 0 \text{ V}$$

c. $$\xi = -L \frac{\Delta I}{\Delta t} = -\left(3.2 \times 10^{-3} \text{ H}\right) \left(\frac{0 \text{ A} - 4.0 \text{ A}}{9.0 \times 10^{-3} \text{ s} - 5.0 \times 10^{-3} \text{ s}}\right) = +3.2 \text{ V}$$

51. **REASONING** We will designate the coil containing the current as the primary coil, and the other as the secondary coil. The emf $\xi_s$ induced in the secondary coil due to the changing current in the primary coil is given by $\xi_s = -M \left(\frac{\Delta I_p}{\Delta t}\right)$ (Equation 22.7), where $M$ is the mutual inductance of the two coils, $\Delta I_p$ is the change in the current in the primary coil, and $\Delta t$ is the change in time. This equation can be used to find the mutual inductance.

**SOLUTION** Solving Equation 22.7 for the mutual inductance gives

$$M = -\frac{\xi_s \Delta t}{\Delta I_p} = -\left(\frac{1.7 \text{ V}}{3.7 \times 10^{-2} \text{ s}}\right) = 4.5 \times 10^{-2} \text{ H}$$

52. **REASONING** According to $\xi_s = -M \frac{\Delta I_p}{\Delta t}$ (Equation 22.7), a change $\Delta I_p$ in the current in the primary coil induces an emf $\xi_s$ in the secondary, where $M$ is the mutual inductance of the two coils and $\Delta t$ is the time interval of the current change. We are interested only in the magnitude of the current change $\Delta I_p$, so we will omit the minus sign in Equation 22.7. The induced emf $\xi_s$ in the secondary coil will drive a current $I_s$, as we see from Ohm’s law: $\xi_s = I_s R$ (Equation 20.2), where $R$ is the resistance of the circuit that includes the secondary coil.
**SOLUTION** Omitting the minus sign in \( \xi_s = -M \frac{\Delta I_p}{\Delta t} \) (Equation 22.7) and solving for \( \Delta I_p \) yields

\[
\Delta I_p = \frac{\xi_s \Delta t}{M}
\]

Substituting \( \xi_s = I_s R \) (Equation 20.2) into Equation (1), we obtain

\[
\Delta I_p = \frac{\xi_s \Delta t}{M} = \frac{I_s R \Delta t}{M} = \frac{(6.0 \times 10^{-3} \text{ A})(12 \text{ } \Omega)(72 \times 10^{-3} \text{ s})}{3.2 \times 10^{-3} \text{ H}} = 1.6 \text{ A}
\]

53. **REASONING AND SOLUTION** The induced emf in the secondary coil is proportional to the mutual inductance. If the primary coil is assumed to be unaffected by the metal, that is \( \Delta I_1/\Delta t \) is the same for both cases, then

New emf = 3(0.46 V) = 1.4 V

54. **REASONING** According to Faraday’s law of electromagnetic induction, expressed as \( \xi = -L \frac{\Delta I}{\Delta t} \) (Equation 22.9), an emf is induced in the solenoid as long as the current is changing in time.

The amount of electrical energy \( E \) stored by an inductor is \( E = \frac{1}{2} L I^2 \) (Equation 22.10), where \( L \) is the inductance and \( I \) is the current.

The power \( P \) is equal to the energy removed divided by the time \( t \) or \( P = \frac{E}{t} = \frac{1}{2} L I^2 \) (Equation 6.10b).

**SOLUTION**

a. The emf induced in the solenoid is

\[
\text{Emf} = -L \left( \frac{\Delta I}{\Delta t} \right) = -(3.1 \text{ H}) \left( \frac{0 \text{ A} - 15 \text{ A}}{75 \times 10^{-3} \text{ s}} \right) = +620 \text{ V}
\]

b. The energy stored in the solenoid is

\[
E = \frac{1}{2} L I^2 = \frac{1}{2} (3.1 \text{ H})(15 \text{ A})^2 = 350 \text{ J}
\]

c. The rate (or power, \( P \)) at which the energy is removed is

\[
P = \frac{E}{t} = \frac{\frac{1}{2} L I^2}{t} = \frac{\frac{1}{2} (3.1 \text{ H})(15\text{A})^2}{75 \times 10^{-3} \text{ s}} = 4700 \text{ W}
\]
55. **REASONING AND SOLUTION** From the results of Example 13, the self-inductance \( L \) of a long solenoid is given by \( L = \mu_0 n^2 A \ell \). Solving for the number of turns \( n \) per unit length gives

\[
n = \frac{L}{\mu_0 A \ell} = \frac{1.4 \times 10^{-3} \text{ H}}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.2 \times 10^{-3} \text{ m}^2)(0.052 \text{ m})} = 4.2 \times 10^3 \text{ turns/m}
\]

Therefore, the total number of turns \( N \) is the product of \( n \) and the length \( \ell \) of the solenoid:

\[
N = n \ell = (4.2 \times 10^3 \text{ turns/m})(0.052 \text{ m}) = 220 \text{ turns}
\]

56. **REASONING** Let us arbitrarily label the long solenoid as the primary (p) and the coil as the secondary (s). The mutual inductance \( M \) is \( M = \frac{N_s \Phi_s}{I_p} \) (Equation 22.6), where \( N_s = 125 \) is the number of turns in the coil, \( \Phi_s \) is the flux through one loop of the coil due to the current \( I_p \) in the solenoid. Since the flux \( \Phi_s \) is due to the current in the solenoid, the flux is \( \Phi_s = B_{\text{solenoid}} A \cos \phi \) (Equation 22.3), where \( A = \pi R^2 \) is the area of a turn of the coil (or the solenoid) and \( \phi = 0^\circ \) is the angle between the normal to the coil and the field of the solenoid. Note that a turn of the solenoid and a turn of the coil have the virtually the same radius \( R = 0.0180 \text{ m} \). The field of the solenoid is \( B_{\text{solenoid}} = \mu_0 n I_p \) (Equation 21.7), where \( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \) is the permeability of free space, \( n = 1750 \text{ m}^{-1} \) is the number of turns per meter in the solenoid, and \( I_p \) is the current in the solenoid. We do not have a value for the current \( I_p \) in the solenoid. However, we will see that it will be eliminated algebraically from the solution.

**SOLUTION** Substituting \( \Phi_s = B_{\text{solenoid}} A \cos \phi \) (Equation 22.3) into Equation 22.6 for the mutual inductance gives

\[
M = \frac{N_s \Phi_s}{I_p} = \frac{N_s (B_{\text{solenoid}} A \cos \phi)}{I_p} = \frac{N_s (B_{\text{solenoid}} \pi R^2 \cos 0^\circ)}{I_p} = \frac{N_s (B_{\text{solenoid}} \pi R^2)}{I_p} \tag{22.6}
\]

Substituting \( B_{\text{solenoid}} = \mu_0 n I_p \) (Equation 21.7) into this result for \( M \), we obtain

\[
M = \frac{N_s (B_{\text{solenoid}} \pi R^2)}{I_p} = \frac{N_s (\mu_0 n I_p) \pi R^2}{I_p} = N_s \mu_0 n \pi R^2 \tag{22.6}
\]

\[
= 125(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1750 \text{ m}^{-1}) \pi (0.0180 \text{ m})^2 = 2.80 \times 10^{-4} \text{ H}
\]
57. **REASONING** As demonstrated in Example 13, the inductance \( L \) of a solenoid of length \( \ell \) and cross-sectional area \( A \) is given by

\[
L = \mu_0 n^2 \ell A
\]

(1)

where \( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \) is the permeability of free space and \( n \) is the number of turns per unit length. It may appear that solving Equation (1) for the length \( \ell \) of the solenoid would determine \( \ell \) as a function of the inductance \( L \), and therefore solve the problem. However, the number \( n \) of turns per unit length is the ratio of the total number \( N \) of turns to the length \( \ell \):

\[
n = \frac{N}{\ell}
\]

(2)

Therefore, changing the length \( \ell \) of the solenoid also changes the number \( n \) of turns per unit length, since the total number \( N \) of turns is unchanged. Using Equations (1) and (2), we will derive an expression for the length of the solenoid in terms of its inductance \( L \) and the constant quantities \( N, \mu_0, \) and \( A \).

**SOLUTION** Substituting Equation (2) into Equation (1) and simplifying, we obtain

\[
L = \mu_0 n^2 \ell A = \mu_0 \left( \frac{N}{\ell} \right)^2 \ell A = \frac{\mu_0 N^2 A}{\ell}
\]

(3)

Solving Equation (3) for the length of the solenoid yields

\[
\ell = \frac{\mu_0 N^2 A}{L}
\]

(4)

When the solenoid’s length is \( \ell_1 \), its inductance is \( L_1 = 5.40 \times 10^{-5} \text{ H} \). Reducing its length to \( \ell_2 \) increases its inductance to \( L_2 = 8.60 \times 10^{-5} \text{ H} \). The amount \( \Delta \ell = \ell_1 - \ell_2 \) by which the solenoid’s length decreases is, from Equation (4),

\[
\Delta \ell = \ell_1 - \ell_2 = \frac{\mu_0 N^2 A}{L_1} - \frac{\mu_0 N^2 A}{L_2} = \mu_0 N^2 A \left( \frac{1}{L_1} - \frac{1}{L_2} \right)
\]

\[
= \left( 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \right) \left( 65 \right)^2 \left( 9.0 \times 10^{-4} \text{ m}^2 \right) \left( \frac{1}{5.40 \times 10^{-5} \text{ H}} - \frac{1}{8.60 \times 10^{-5} \text{ H}} \right) = 0.033 \text{ m}
\]

58. **REASONING** The emf due to self-induction is given by \( \xi = -L \left( \Delta I / \Delta t \right) \) (Equation 22.9), where \( L \) is the self-inductance, and \( \Delta I / \Delta t \) is the rate at which the current changes. Both \( \Delta I \) and \( \Delta t \) are known. We can find the self-inductance of a toroid by starting with that of a long solenoid, which we obtained in Example 13.

**SOLUTION** The emf induced in the toroid is given by

\[
\xi = -L \left( \frac{\Delta I}{\Delta t} \right)
\]

(22.9)
As obtained in Example 13, the self-inductance of a long solenoid is \( L = \mu_0 n^2 A \ell \), where \( \ell \) is the length of the solenoid, \( n \) is the number of turns per unit length, and \( A \) is the cross-sectional area of the solenoid. A toroid is a solenoid that is bent to form a circle of radius \( R \). The length \( \ell \) of the toroid is the circumference of the circle, \( \ell = 2\pi R \). Substituting this expression for \( \ell \) into the equation for \( L \), we obtain
\[
L = \mu_0 n^2 A \ell = \mu_0 n^2 A (2\pi R)
\]
Substituting this relation for \( L \) into Equation (22.9) yields
\[
\xi = -L \left( \frac{\Delta I}{\Delta t} \right) = -\mu_0 n^2 A (2\pi R) \left( \frac{\Delta I}{\Delta t} \right)
\]
\[
= -(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (2400 \text{ m}^{-1})^2 (1.0 \times 10^{-6} \text{ m}^2) 2\pi(0.050 \text{ m}) \left( \frac{1.1 \text{ A} - 2.5 \text{ A}}{0.15 \text{ s}} \right)
\]
\[
= 2.1 \times 10^{-5} \text{ V}
\]

59. **REASONING AND SOLUTION** The mutual inductance is
\[
M = \frac{N_2 \Phi_2}{I_1}
\]
The flux through loop 2 is
\[
\Phi_2 = B_1 A_2 = \left( \frac{\mu_0 N_1 I_1}{2R_1} \right) (\pi R_2^2)
\]
Then
\[
M = \frac{N_2 \Phi_2}{I_1} = \frac{N_2}{I_1} \left( \frac{\mu_0 N_1 I_1}{2R_1} \right) (\pi R_2^2) = \frac{\mu_0 \pi N_1 N_2 R_2^2}{2R_1}
\]

60. **REASONING** To solve this problem, we can use the transformer equation: \( \frac{V_s}{V_p} = \frac{N_s}{N_p} \) (Equation 22.12), where \( V_s \) is the voltage provided by the secondary coil, \( V_p = 120 \text{ V} \) is the voltage applied across the primary coil, and \( \frac{N_s}{N_p} = \frac{1}{32} \) is the turns ratio.

**SOLUTION** Solving the transformer equation for the voltage \( V_s \) provided by the secondary coil gives
\[
\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \text{or} \quad V_s = V_p \frac{N_s}{N_p} = (120 \text{ V}) \left( \frac{1}{32} \right) = 3.8 \text{ V}
\]
61. **REASONING** The air filter is connected to the secondary, so that the power used by the air filter is the power provided by the secondary. However, the power provided by the secondary comes from the primary, so the power used by the air filter is also the power delivered by the wall socket to the primary. This power is \( \bar{P} = I_p V_p \) (Equation 20.15a), where \( I_p \) is the current in the primary and \( V_p \) is the voltage provided by the socket, which we know. Although we do not have a value for \( I_p \), we do have a value for \( I_s \), which is the current in the secondary. We will take advantage of the fact that \( I_p \) and \( I_s \) are related according to
\[
\frac{I_s}{I_p} = \frac{N_p}{N_s} \quad \text{(Equation 22.13)}.
\]

**SOLUTION** The power used by the filter is
\[
\bar{P} = I_p V_p
\]

Solving Equation 22.13 for \( I_p \) shows that
\[
I_p = I_s \left( \frac{N_s}{N_p} \right)
\]
Substituting this result into the expression for the power gives
\[
\bar{P} = I_p V_p = I_s \left( \frac{N_s}{N_p} \right) V_p = \left( 1.7 \times 10^{-3} \text{ A} \right) \left( \frac{50}{1} \right) (120 \text{ V}) = 1.0 \times 10^1 \text{ W}
\]

62. **REASONING** Since the secondary voltage (the voltage to charge the batteries) is less than the primary voltage (the voltage at the wall socket), the transformer is a step-down transformer.

In a step-down transformer, the voltage across the secondary coil is less than the voltage across the primary coil. However, the current in the secondary coil is greater than the current in the primary coil. Thus, the current that goes through the batteries is greater than the current from the wall socket.

If the transformer has negligible resistance, the power delivered to the batteries is equal to the power coming from the wall socket.

**SOLUTION**

a. The turns ratio \( N_s / N_p \) is equal to the ratio of secondary voltage to the primary voltage:
\[
\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{9.0 \text{ V}}{120 \text{ V}} = 1:13
\]

b. The current from the wall socket is given by Equation 22.13:
\[
I_p = I_s \left( \frac{N_s}{N_p} \right) = \left( 225 \times 10^{-3} \text{ A} \right) \left( \frac{1}{13} \right) = 1.7 \times 10^{-2} \text{ A}
\]
c. The average power delivered by the wall socket is the product of the primary current and voltage:

\[ P_p = I_p V_p = (17 \times 10^{-3} \text{ A})(120 \text{ V}) = 2.0 \text{ W} \]  

(20.15a)

The average power delivered to the batteries is the same as that coming from the wall socket, so \( P_s = 2.0 \text{ W} \).

63. **REASONING AND SOLUTION** The resistance of the primary is (see Equation 20.3)

\[ R_p = \frac{\rho L_p}{A} \]  

(1)

The resistance of the secondary is

\[ R_s = \frac{\rho L_s}{A} \]  

(2)

In writing Equations (1) and (2) we have used the fact that both coils are made of the same wire, so that the resistivity \( \rho \) and the cross-sectional area \( A \) is the same for each. Division of the equations gives

\[ \frac{R_s}{R_p} = \frac{L_s}{L_p} = \frac{14 \Omega}{56 \Omega} = \frac{1}{4} \]

Since the diameters of the coils are the same, the lengths of the wires are proportional to the number of turns. Therefore,

\[ \frac{N_s}{N_p} = \frac{L_s}{L_p} = \frac{1}{4} \]

64. **REASONING** The generator drives a fluctuating current in the primary coil, and the changing magnetic flux that results from this current induces a fluctuating voltage in the secondary coil, attached to the resistor. The peak emf of the generator is equal to the peak voltage \( V_p \) of the primary coil. Given a peak voltage in the secondary coil of \( V_s = 67 \text{ V} \), the peak voltage \( V_p \) can be found from

\[ \frac{V_s}{V_p} = \frac{N_s}{N_p} \]  

(22.12)

In Equation 22.12, \( N_p \) and \( N_s \) are, respectively, the number of turns in the primary coil and the number of turns in the secondary coil.

**SOLUTION** Solving Equation 22.12 for \( V_p \), we obtain

\[ V_p = V_s \left( \frac{N_p}{N_s} \right) \]  

(1)
Since there are $N_p = 11$ turns in the primary coil and $N_s = 18$ turns in the secondary coil, Equation (1) gives

$$V_p = (67 \text{ V}) \left(\frac{11}{18}\right) = 41 \text{ V}$$

65. **REASONING** The ratio $(I_s / I_p)$ of the current in the secondary coil to that in the primary coil is equal to the ratio $(N_p / N_s)$ of the number of turns in the primary coil to that in the secondary coil. This relation can be used directly to find the current in the primary coil.

**SOLUTION** Solving the relation $(I_s / I_p) = (N_s / N_p)$ (Equation 22.13) for $I_p$ gives

$$I_p = I_s \left(\frac{N_s}{N_p}\right) = (1.6 \text{ A}) \left(\frac{1}{8}\right) = 0.20 \text{ A}$$

66. **REASONING** The Turns ratio is related to the current $I_p$ in the primary and the current $I_s$ in the secondary according to $I_p = I_s \frac{N_s}{N_p}$ (Equation 22.13).

The turns ratio is related to the voltage $V_p$ in the primary and the voltage $V_s$ in the secondary according to the transformer equation, which is $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ (Equation 22.12).

Given a value for the power $P$ used by the picture tube means that we know the power provided by the secondary. Since power is current times voltage (Equation 20.15a), we know that $P = I_s V_s$. This expression can be solved for $V_s$, and the result can be substituted into Equation 22.12 to give the turns ratio:

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{P}{I_s V_P} = \frac{P}{I_s V_P}$$

**SOLUTION** Using Equation (1), we find that the turns ratio is

$$\frac{N_s}{N_p} = \frac{P}{I_s V_P} = \frac{91 \text{ W}}{(5.5 \times 10^{-3} \text{ A})(120 \text{ V})} = 140$$

67. **REASONING** The power used to heat the wires is given by Equation 20.6b:

$$P = I^2 R$$. Before we can use this equation, however, we must determine the total resistance $R$ of the wire and the current that flows through the wire.
**SOLUTION**

a. The total resistance of one of the wires is equal to the resistance per unit length multiplied by the length of the wire. Thus, we have

\[(5.0 \times 10^{-2} \text{ } \Omega/\text{km})(7.0 \text{ } \text{km}) = 0.35 \text{ } \Omega\]

and the total resistance of the transmission line is twice this value or 0.70 \text{ } \Omega. According to Equation 20.6a \((P = IV)\), the current flowing into the town is

\[I = \frac{P}{V} = \frac{1.2 \times 10^6 \text{ } \text{W}}{1200 \text{ } \text{V}} = 1.0 \times 10^3 \text{ } \text{A}\]

Thus, the power used to heat the wire is

\[P = I^2R = \left(1.0 \times 10^3 \text{ } \text{A}\right)^2(0.70 \text{ } \Omega) = 7.0 \times 10^5 \text{ } \text{W}\]

b. According to the transformer equation (Equation 22.12), the stepped-up voltage is

\[V_s = V_p \left(\frac{N_s}{N_p}\right) = (1200 \text{ } \text{V})\left(\frac{100}{1}\right) = 1.2 \times 10^5 \text{ } \text{V}\]

According to Equation 20.6a \((P = IV)\), the current in the wires is

\[I = \frac{P}{V} = \frac{1.2 \times 10^6 \text{ } \text{W}}{1.2 \times 10^5 \text{ } \text{V}} = 1.0 \times 10^1 \text{ } \text{A}\]

The power used to heat the wires is now

\[P = I^2R = \left(1.0 \times 10^1 \text{ } \text{A}\right)^2(0.70 \text{ } \Omega) = 7.0 \times 10^1 \text{ } \text{W}\]

68. **REASONING** The power that your house is using can be determined from \(\bar{P} = I_{\text{rms}}\xi_\text{rms}\) (Equation 20.15a), where \(I_{\text{rms}}\) is the current in your house. We know that \(\xi_\text{rms}\) is 240 V. To find \(I_{\text{rms}}\), we must apply \(\frac{I_s}{I_p} = \frac{N_p}{N_s}\) (Equation 22.13) to convert the current given for the primary of the substation transformer into the current in the secondary of the substation transformer. This secondary current then becomes the current in the primary of the transformer on the telephone pole. We will use Equation 22.13 again to find the current in the secondary of the transformer on the telephone pole. This secondary current is \(I_{\text{rms}}\).

**SOLUTION** According to Equation 20.15a, the power that your house is using is

\[\bar{P} = I_{\text{rms}}\xi_\text{rms}\]  \hspace{2cm} (1)

Applying Equation 22.13 to the substation transformer, we find
\[ \frac{I_S}{I_P} = \frac{N_P}{N_S} \quad \text{or} \quad I_S = I_P \left( \frac{N_P}{N_S} \right) = (48 \times 10^{-3} \text{ A}) \left( \frac{29}{1} \right) = 1.4 \text{ A} \]

Thus, the current in the primary of the transformer on the telephone pole is 1.4 A. Applying Equation 22.3 to the telephone-pole transformer, we obtain

\[ \frac{I_S}{I_P} = \frac{N_P}{N_S} \quad \text{or} \quad I_S = I_P \left( \frac{N_P}{N_S} \right) = (1.4 \text{ A}) \left( \frac{32}{1} \right) = 45 \text{ A} \]

Using this value of 45 A for the current in Equation (1) gives

\[ \bar{P} = I_{\text{rms}} \bar{V}_{\text{rms}} = (45 \text{ A})(240 \text{ V}) = 1.1 \times 10^4 \text{ W} \]

69. **REASONING AND SOLUTION** Ohm’s law written for the secondary is \( V_s = I_s R_2 \). We know that

\[ V_s = (N_s / N_p) V_p \quad \text{and} \quad I_s = (N_p / N_s) I_p \]

Substituting these expressions for \( V_s \) and \( I_s \) into \( V_s = I_s R_2 \) and recognizing that \( R_1 = V_p / I_p \), we find that

\[ R_1 = \left( \frac{N_p}{N_s} \right)^2 R_2 \]

70. **REASONING** According to Equation 22.3, Faraday’s law specifies the emf induced in a coil of \( N \) loops as

\[ \varepsilon = -N \frac{\Delta \Phi}{\Delta t} \]

where \( \Delta \Phi / \Delta t \) is the rate of change of the magnetic flux in a single loop. Recognizing that \( \Delta \Phi / \Delta t \) is the same for each of the coils, we will apply Faraday’s law to each coil to obtain our solution.

**SOLUTION** Applying Faraday’s law to each coil gives

\[ \varepsilon_1 = -N_1 \frac{\Delta \Phi}{\Delta t} \quad \text{and} \quad \varepsilon_2 = -N_2 \frac{\Delta \Phi}{\Delta t} \]

Dividing these equations and remembering that the rate of change of the flux is the same for each coil, we find that

\[ \frac{\varepsilon_2}{\varepsilon_1} = -N_2 \frac{\Delta \Phi}{\Delta t} = \frac{N_2}{N_1} \quad \text{or} \quad N_2 = N_1 \frac{\varepsilon_2}{\varepsilon_1} = 184 \left( \frac{4.23 \text{ V}}{2.82 \text{ V}} \right) = 276 \]
71. **REASONING** The peak emf \( \xi_0 \) of a generator is found from \( \xi_0 = NAB\omega \) (Equation 22.4), where \( N \) is the number of turns in the generator coil, \( A \) is the coil’s cross-sectional area, \( B \) is the magnitude of the uniform magnetic field in the generator, and \( \omega \) is the angular frequency of rotation of the coil. In terms of the frequency \( f \) (in Hz), the angular frequency is given by \( \omega = 2\pi f \) (Equation 10.6). Substituting Equation 10.6 into Equation 22.4, we obtain

\[
\xi_0 = NAB(2\pi f) = 2\pi NABf
\]

(1)

When the rotational frequency \( f \) of the coil changes, the peak emf \( \xi_0 \) also changes. The quantities \( N, A, \) and \( B \) remain constant, however, because they depend on how the generator is constructed, not on how rapidly the coil rotates. We know the peak emf of the generator at one frequency, so we will use Equation (1) to determine the peak emf for a different frequency in part (a), and the frequency needed for a different peak emf in part (b).

**SOLUTION**

a. Solving Equation (1) for the quantities that do not change with frequency, we find that

\[
\frac{\xi_0}{f} = \frac{2\pi NAB}{\text{Same for all frequencies}}
\]

(2)

The peak emf is \( \xi_{0,1} = 75 \text{ V} \) when the frequency is \( f_1 = 280 \text{ Hz} \). We wish to find the peak emf \( \xi_{0,2} \) when the frequency is \( f_2 = 45 \text{ Hz} \). From Equation (2), we have that

\[
\frac{\xi_{0,2}}{f_2} = \frac{2\pi NAB}{\text{Same for all frequencies}} = \frac{\xi_{0,1}}{f_1}
\]

(3)

Solving Equation (3) for \( \xi_{0,2} \), we obtain

\[
\xi_{0,2} = \left( \frac{f_2}{f_1} \right) \xi_{0,1} = \left( \frac{45 \text{ Hz}}{280 \text{ Hz}} \right)(75 \text{ V}) = 12 \text{ V}
\]

b. Letting \( \xi_{0,3} = 180 \text{ V} \), Equation (2) yields

\[
\frac{\xi_{0,3}}{f_3} = \frac{\xi_{0,1}}{f_1}
\]

(4)

Solving Equation (4) for \( f_3 \), we find that

\[
f_3 = \left( \frac{\xi_{0,3}}{\xi_{0,1}} \right) f_1 = \left( \frac{180 \text{ V}}{75 \text{ V}} \right)(280 \text{ Hz}) = 670 \text{ Hz}
\]
72. **REASONING** According to Faraday's law, as given in Equation 22.3, the magnitude of the emf is \[|\xi| = -\left(1\right) \frac{\Delta \Phi}{\Delta t}\], where we have set \(N = 1\) for a single turn. Since the normal is parallel to the magnetic field, the angle \(\phi\) between the normal and the field is \(\phi = 0^\circ\) when calculating the flux \(\Phi\) from Equation 22.2: \(\Phi = BA\cos 0^\circ = BA\). We will use this expression for the flux in Faraday's law.

**SOLUTION** Representing the flux as \(\Phi = BA\), we find that the magnitude of the induced emf is

\[|\xi| = -\frac{\Delta \Phi}{\Delta t} = -\frac{\Delta (BA)}{\Delta t} = \frac{B \Delta A}{\Delta t}\]

In this result we have used the fact that the field magnitude \(B\) is constant. Rearranging this equation gives

\[\frac{\Delta A}{\Delta t} = \frac{|\xi|}{B} = \frac{2.6 \text{ V}}{1.7 \text{ T}} = 1.5 \text{ m}^2/\text{s}\]

73. **SSM REASONING** In solving this problem, we apply Lenz's law, which essentially says that the change in magnetic flux must be opposed by the induced magnetic field.

**SOLUTION**

a. The magnetic field due to the wire in the vicinity of position 1 is directed out of the paper. The coil is moving closer to the wire into a region of higher magnetic field, so the flux through the coil is increasing. Lenz's law demands that the induced field counteract this increase. The direction of the induced field, therefore, must be into the paper. The current in the coil must be **clockwise**.

b. At position 2 the magnetic field is directed into the paper and is decreasing as the coil moves away from the wire. The induced magnetic field, therefore, must be directed into the paper, so the current in the coil must be **clockwise**.

74. **REASONING AND SOLUTION** According to the transformer equation (Equation 22.12), we have

\[N_s = \left(\frac{V_s}{V_p}\right) N_p = \left(\frac{4320 \text{ V}}{120.0 \text{ V}}\right) (21) = 756\]

75. **SSM REASONING AND SOLUTION** The energy stored in a capacitor is given by Equation 19.11b as \(\frac{1}{2} CV^2\). The energy stored in an inductor is given by Equation 22.10 as \(\frac{1}{2} LI^2\). Setting these two equations equal to each other and solving for the current \(I\), we get
76. REASONING When the motor has just started turning the fan blade, there is no back emf, and the voltage across the resistance \( R \) of the motor is equal to the voltage \( V_0 \) of the outlet. Under this condition the resulting current in the motor is

\[
I_0 = \frac{V_0}{R}
\]

(1)

according to Ohm’s law. When the fan blade is turning at its normal operating speed, the back emf \( \xi \) in the motor reduces the voltage across the resistance \( R \) of the motor to \( V_0 - \xi \), so that, the current drawn by the motor is

\[
I = \frac{V_0 - \xi}{R}
\]

(22.5)

The current \( I \) drawn at the normal operating speed is only 15.0% of the current \( I_0 \) drawn when the fan blade just begins to turn, so we have that

\[
I = 0.150I_0
\]

(2)

SOLUTION Substituting Equation (2) into Equation (22.5) yields

\[
0.150I_0 = \frac{V_0 - \xi}{R}
\]

(3)

Substituting Equation (1) into Equation (3) and solving for \( \xi \), we find that

\[
0.150\left(\frac{V_0}{R}\right) = \frac{V_0 - \xi}{R}
\]

or \( \xi = V_0 - 0.150V_0 = 0.850V_0 = 0.850(120.0 \text{ V}) = 102 \text{ V} \)

77. REASONING Using Equation 22.3 (Faraday’s law) and recognizing that \( N = 1 \), we can write the magnitude of the emfs for parts \( a \) and \( b \) as follows:

\[
|\xi_a| = -\left(\frac{\Delta\Phi}{\Delta t}\right)_a \quad \text{and} \quad |\xi_b| = -\left(\frac{\Delta\Phi}{\Delta t}\right)_b
\]

(1) and

\[
|\xi_a| = \left(\frac{\Delta\Phi}{\Delta t}\right)_a \quad \text{and} \quad |\xi_b| = \left(\frac{\Delta\Phi}{\Delta t}\right)_b
\]

(2)

To solve this problem, we need to consider the change in flux \( \Delta\Phi \) and the time interval \( \Delta t \) for both parts of the drawing in the text.

SOLUTION The change in flux is the same for both parts of the drawing and is given by

\[
(\Delta\Phi)_a = (\Delta\Phi)_b = \Phi_{\text{inside}} - \Phi_{\text{outside}} = \Phi_{\text{inside}} = BA
\]

(3)
In Equation (3) we have used the fact that initially the coil is outside the field region, so that \( \Phi_{\text{outside}} = 0 \) Wb for both cases. Moreover, the field is perpendicular to the plane of the coil and has the same magnitude \( B \) over the entire area \( A \) of the coil, once it has completely entered the field region. Thus, \( \Phi_{\text{inside}} = BA \) in both cases, according to Equation 22.2.

The time interval required for the coil to enter the field region completely can be expressed as the distance the coil travels divided by the speed at which it is pushed. In part \( a \) of the drawing the distance traveled is \( W \), while in part \( b \) it is \( L \). Thus, we have

\[
(\Delta t)_a = \frac{W}{v} \quad (4) \quad (\Delta t)_b = \frac{L}{v} \quad (5)
\]

Substituting Equations (3), (4), and (5) into Equations (1) and (2), we find

\[
|\varepsilon_a| = \left| -\frac{\Delta \Phi}{\Delta t} \right|_a = \frac{BA}{W/v} \quad (6) \quad |\varepsilon_b| = \left| -\frac{\Delta \Phi}{\Delta t} \right|_b = \frac{BA}{L/v} \quad (7)
\]

Dividing Equation (6) by Equation (7) gives

\[
\frac{|\varepsilon_a|}{|\varepsilon_b|} = \frac{BA}{W/v} \cdot \frac{L/v}{BA} = \frac{L}{W} = 3.0 \quad \text{or} \quad \frac{|\varepsilon_a|}{3.0} = \frac{|\varepsilon_b|}{3.0} = \frac{0.15 \text{ V}}{3.0} = 0.050 \text{ V}
\]

78. **REASONING AND SOLUTION** If the applied magnetic field is decreasing in time, then the flux through the circuit is decreasing. Lenz's law requires that an induced magnetic field be produced which attempts to counteract this decrease; hence its direction is out of the paper. The sense of the induced current in the circuit must be CCW. Therefore, the lower plate of the capacitor is positive while the upper plate is negative. The electric field between the plates of the capacitor points from positive to negative so the electric field points **upward**.

79. **SSM REASONING** The energy dissipated in the resistance is given by Equation 6.10b as the power \( P \) dissipated multiplied by the time \( t \), Energy = \( Pt \). The power, according to Equation 20.6c, is the square of the induced emf \( \varepsilon \) divided by the resistance \( R \), \( P = \varepsilon^2/R \). The induced emf can be determined from Faraday's law of electromagnetic induction, Equation 22.3.

**SOLUTION** Expressing the energy consumed as Energy = \( Pt \), and substituting in \( P = \varepsilon^2/R \), we find
Energy = \frac{\varepsilon^2 t}{R}

The induced emf is given by Faraday’s law as \( \varepsilon = -N \frac{\Delta \Phi}{\Delta t} \), and the resistance \( R \) is equal to the resistance per unit length \((3.3 \times 10^{-2} \ \Omega/m)\) times the length of the circumference of the loop, \(2\pi r\). Thus, the energy dissipated is

\[
\text{Energy} = \frac{\left( -N \frac{\Delta \Phi}{\Delta t} \right)^2 \frac{t}{(3.3 \times 10^{-2} \ \Omega/m)2\pi r}}{\left( 3.3 \times 10^{-2} \ \Omega/m \right)2\pi r} = \frac{\left( -N \frac{\Phi - \Phi_0}{t-t_0} \right)^2 \frac{t}{(3.3 \times 10^{-2} \ \Omega/m)2\pi r}}{\left( 3.3 \times 10^{-2} \ \Omega/m \right)2\pi r}
\]

\[
= \frac{\left[ -N \left( \frac{BA \cos \phi - B_0 A \cos \phi}{t-t_0} \right) \right]^2 \frac{t}{(3.3 \times 10^{-2} \ \Omega/m)2\pi r}}{\left( 3.3 \times 10^{-2} \ \Omega/m \right)2\pi r} = \frac{\left[ -NA \cos \phi \left( \frac{B-B_0}{t-t_0} \right) \right]^2 \frac{t}{(3.3 \times 10^{-2} \ \Omega/m)2\pi (0.12 \ m)}}{\left( 3.3 \times 10^{-2} \ \Omega/m \right)2\pi (0.12 \ m)} = 6.6 \times 10^{-2} \ J
\]

80. **REASONING AND SOLUTION** If the rectangle is made \( \Delta I = I_t \) wide, then the top of the rectangle will intersect the line at \( LI_t \). The work is one-half the area of the rectangle, so

\[
W = \frac{1}{2} (LI_t)I_t = \frac{1}{2} LI_t^2
\]

81. **REASONING** According to Ohm’s law, the average current \( I \) induced in the coil is given by \( I = \frac{|\varepsilon|}{R} \), where \( |\varepsilon| \) is the magnitude of the induced emf and \( R \) is the resistance of the coil. To find the induced emf, we use Faraday’s law of electromagnetic induction

**SOLUTION** The magnitude of the induced emf can be found from Faraday’s law of electromagnetic induction and is given by Equation 22.3:

\[
|\varepsilon| = -N \frac{\Delta \Phi}{\Delta t} = -N \frac{\Delta (BA \cos 0^\circ)}{\Delta t} = NA \frac{\Delta B}{\Delta t}
\]

We have used the fact that the field within a long solenoid is perpendicular to the cross-sectional area \( A \) of the solenoid and makes an angle of \( 0^\circ \) with respect to the normal to the
area. The field is given by Equation 21.7 as \( B = \mu_0 n I \), so the change \( \Delta B \) in the field is \( \Delta B = \mu_0 n \Delta I \), where \( \Delta I \) is the change in the current. The induced current is, then,

\[
I = \frac{N A \Delta B}{R} = \frac{N A \mu_0 n \Delta I}{R} = \frac{(10)(6.0 \times 10^{-4} \text{ m}^2)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(400 \text{ turns/m})(0.40 \text{ A})}{(0.050 \text{ s})} = 1.6 \times 10^{-5} \text{ A}
\]

82. **REASONING** The generator coil rotates in an external magnetic field \( B \), inducing a varying emf with a peak value \( \xi_0 \) and a varying current \( I \) with a peak value \( I_0 \). Once the current is flowing, the external magnetic field \( B \) exerts a torque \( \tau \) on the coil, according to

\[
\tau = NIAB \sin \phi \tag{21.4}
\]

In Equation 21.4, \( N \) is the number of turns in the generator coil, \( A \) is its cross-sectional area, and \( \phi \) is the angle between the external magnetic field and the normal to the plane of the coil. Following Lenz's law, this torque opposes the rotation that induces the emf and the current, hence it is labeled a countertorque. The induced emf is given by

\[
\xi = N A B \omega \sin \omega t = \xi_0 \sin \omega t \tag{22.4}
\]

where \( \omega \) is the angular frequency of the rotation of the coil.

In terms of the peak current \( I_0 \) and peak voltage \( V_0 \) across the light bulb, the average power \( \bar{P} \) that the generator delivers to the light bulb is \( \bar{P} = \frac{1}{2} I_0 V_0 \) (Equation 20.10). The peak voltage \( V_0 \) across the bulb is equal to the peak emf \( \xi_0 \) of the generator, and we have that

\[
\bar{P} = \frac{1}{2} I_0 V_0 = \frac{1}{2} I_0 \xi_0 \tag{20.10}
\]

**SOLUTION** The peak value \( \xi_0 \) of the induced emf occurs when \( \sin \omega t = 1 \) in Equation 22.4, which is the instant when the plane of the coil is parallel to the external magnetic field. At this instant, \( \phi = 90^\circ \) in Equation 21.4. Because the peak value \( I_0 \) of the current occurs at the same instant as the peak value \( \xi_0 = N A B \omega \) of the emf, the maximum value \( \tau_0 \) of the countertorque is

\[
\tau_0 = NI_0 AB \sin 90^\circ = NI_0 AB \tag{1}
\]
Solving Equation (20.10) for \( I_0 \), we obtain

\[
I_0 = \frac{2\bar{P}}{\xi_0}
\]  

(2)

Substituting Equation (2) into Equation (1) yields

\[
\tau_0 = NAB \left( \frac{2\bar{P}}{\xi_0} \right) = \frac{2\bar{P}NAB}{\xi_0}
\]  

(3)

Substituting \( \xi_0 = NAB\omega \) from Equation 22.4 into Equation (3), we obtain

\[
\tau_0 = \frac{2\bar{P}NAB}{NAB\omega} = \frac{2\bar{P}}{\omega}
\]  

(4)

We are given the frequency \( f = 60.0 \text{ Hz} \), which is related to the angular frequency by \( \omega = 2\pi f \) (Equation 10.6). Making this substitution into Equation (4) gives the maximum countertorque:

\[
\tau_0 = \frac{2\bar{P}}{\omega} = \frac{2\bar{P}}{2\pi f} = \frac{\bar{P}}{\pi f} = \frac{75 \text{ W}}{\pi (60.0 \text{ Hz})} = \frac{0.40 \text{ N} \cdot \text{m}}{}
\]
CHAPTER 23 | ALTERNATING CURRENT CIRCUITS

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (d) According to $\bar{P} = V_{\text{rms}}^2 / R$ (Equation 20.15c), the average power is proportional to the square of the rms voltage. Tripling the voltage causes the power to increase by a factor of $3^2 = 9$.

2. $I_{\text{rms}} = 1.9 \, \text{A}$

3. (b) The current $I_{\text{rms}}$ through a capacitor depends inversely on the capacitive reactance $X_C$, as expressed by the relation $I_{\text{rms}} = V_{\text{rms}} / X_C$ (Equation 23.1). The capacitive reactance becomes infinitely large as the frequency goes to zero (see Equation 23.2), so the current goes to zero.

4. (e) According to $X_C = 1/(2\pi f C)$ (Equation 23.2) and $X_L = 2\pi f L$ (Equation 23.4), doubling the frequency $f$ causes $X_C$ to decrease by a factor of 2 and $X_L$ to increase by a factor of 2.

5. $I_{\text{rms}} = 1.3 \, \text{A}$

6. (a) The component of the phasor along the vertical axis is $V_0 \sin 2\pi ft$ (see the drawing that accompanies this problem), which is the instantaneous value of the voltage.

7. (b) The instantaneous value of the voltage is the component of the phasor that lies along the vertical axis (see Sections 23.1 and 23.2). This vertical component is greatest in B and least in A, so the ranking is (largest to smallest) B, C, A.

8. (d) In a resistor the voltage and current are in phase. This means that the two phasors are colinear.

9. (c) Power is dissipated by the resistor, as discussed in Section 20.5. On the other hand, the average power dissipated by a capacitor is zero (see Section 23.1).

10. $I_{\text{rms}} = 2.00 \, \text{A}$

11. (a) When the rms voltage across the inductor is greater than that across the capacitor, the voltage across the RCL combination leads the current (see Section 23.3).
12. (d) Since \( I_{rms} = V_{rms}/Z \) (Equation 23.6), the current is a maximum when the impedance \( Z \) is a minimum. The impedance is \( Z = \sqrt{R^2 + (X_L - X_C)^2} \) (Equation 23.7), and it has a minimum value when \( X_C = X_L = 50 \, \Omega \).

13. (c) The inductor has a very small reactance at low frequencies and behaves as if it were replaced by a wire with no resistance. Therefore, the circuit behaves as two resistors, \( R_1 \) and \( R_2 \), connected in parallel. The inductor has a very large reactance at high frequencies and behaves as if it were cut out of the circuit, leaving a gap in the connecting wires. The circuit behaves as a single resistance \( R_2 \) connected across the generator. The situation at low frequency gives rise to the largest possible current, because the effective resistance of the parallel combination is smaller than the resistance \( R_2 \).

14. (a) The capacitor has a very small reactance at high frequencies and behaves as if it were replaced by a wire with no resistance. Therefore, the circuit behaves as two resistors, \( R_1 \) and \( R_2 \), connected in parallel. The capacitor has a very large reactance at low frequencies and behaves as if it were cut out of the circuit, leaving a gap in the connecting wires. Therefore, the circuit behaves as a single resistor \( R_1 \) connected across the generator. The situation at high frequencies gives rise to the largest possible current, because the effective resistance of the parallel combination is smaller than the resistance \( R_1 \).

15. (e) At low frequencies, the capacitor has a very large reactance. In the series circuit, this large reactance gives rise to a large impedance and, hence, a small current. The parallel circuit has the larger current, because current can flow through the inductor, which has a small reactance at low frequencies.

16. (b) The resonant frequency \( f_0 \) is given by \( f_0 = \frac{1}{2\pi\sqrt{LC}} \) (Equation 23.10). It depends only on \( C \) and \( L \), and not on \( R \).

17. \( f_0 = 1.3 \times 10^3 \, \text{Hz} \)

18. (d) The resonant frequency \( f_0 \) is given by \( f_0 = \frac{1}{2\pi\sqrt{LC}} \) (Equation 23.10). When a second capacitor is added in parallel, the equivalent capacitance increases (see Section 20.12). Therefore, the resonant frequency decreases.
CHAPTER 23  |  ALTERNATING CURRENT CIRCUITS

PROBLEMS

1. **SSM REASONING** As the frequency $f$ of the generator increases, the capacitive reactance $X_C$ of the capacitor decreases, according to $X_C = \frac{1}{2\pi f C}$ (Equation 23.2), where $C$ is the capacitance of the capacitor. The decreasing capacitive reactance leads to an increasing rms current $I_{rms}$, as we see from $I_{rms} = \frac{V_{rms}}{X_C}$ (Equation 23.1), where $V_{rms}$ is the constant rms voltage across the capacitor. We know that $V_{rms}$ is constant, because it is equal to the constant rms generator voltage. The fuse is connected in series with the capacitor, so both have the same current $I_{rms}$. We will use Equations 23.1 and 23.2 to determine the frequency $f$ at which the rms current is 15.0 A.

**SOLUTION** Solving Equation 23.2 for $f$, we obtain

$$f = \frac{1}{2\pi C X_C}$$  \hspace{1cm} (1)

In terms of the rms current and voltage, Equation 23.1 gives the capacitive reactance as $X_C = \frac{V_{rms}}{I_{rms}}$. Substituting this relation into Equation (1) yields

$$f = \frac{1}{2\pi C \frac{V_{rms}}{I_{rms}}} = \frac{I_{rms}}{2\pi C V_{rms}} = \frac{15.0 \text{ A}}{2\pi \left(63.0 \times 10^{-6} \text{ F}\right)\left(4.00 \text{ V}\right)} = [9470 \text{ Hz}]$$

2. **REASONING** When two capacitors of capacitance $C$ are connected in parallel, their equivalent capacitance $C_p$ is given by $C_p = C + C = 2C$ (Equation 20.18). We will find the capacitive reactance $X_C$ of the equivalent capacitance from $X_C = \frac{1}{2\pi f C_p}$ (Equation 23.2). Because the capacitors are the only devices connected to the generator, the rms voltage $V_{rms}$ across the capacitors is equal to the rms voltage of the generator. Therefore, the capacitive reactance $X_C$ is related to the rms voltage of the generator and the rms current $I_{rms}$ in the circuit by $V_{rms} = I_{rms} X_C$ (Equation 23.1).
**SOLUTION**  Substituting $C_p = 2C$ into $X_C = \frac{1}{2\pi f C_p}$ (Equation 23.2) and solving for $C$, we obtain

$$X_C = \frac{1}{2\pi f C_p} = \frac{1}{2\pi f (2C)} = \frac{1}{4\pi f C}$$

or

$$C = \frac{1}{4\pi f X_C}$$

(1)

Solving $V_{\text{rms}} = I_{\text{rms}} X_C$ (Equation 23.1) for $X_C$ yields $X_C = \frac{V_{\text{rms}}}{I_{\text{rms}}}$. Substituting this result into Equation (1), we obtain

$$C = \frac{1}{4\pi f X_C} = \frac{1}{4\pi f \left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)} = \frac{I_{\text{rms}}}{4\pi f V_{\text{rms}}}. \text{ For } \frac{0.16 \text{ A}}{4\pi (610 \text{ Hz})(24 \text{ V})} = 8.7 \times 10^{-7} \text{ F}$$

3. **REASONING** The reactance $X_C$ of a capacitor is given as $X_C = \frac{1}{2\pi f C}$ (Equation 23.2), where $f$ is the frequency of the ac current and $C$ is the capacitance of the capacitor. We note that $X_C$ is inversely proportional to $f$ for a given value of $C$. Therefore, we will be able to solve this problem by applying Equation 23.2 twice, once for each value of the frequency and each time with the same value of the capacitance.

**SOLUTION** Applying Equation 23.2 for each value of the frequency, we obtain

$$X_{C, 870} = \frac{1}{2\pi f_{870} C} \quad \text{and} \quad X_{C, 460} = \frac{1}{2\pi f_{460} C}$$

Dividing the equation on the left by the equation on the right and noting that the unknown capacitance $C$ is eliminated algebraically, we find that

$$\frac{X_{C, 870}}{X_{C, 460}} = \frac{2\pi f_{870} C}{2\pi f_{460} C} = \frac{f_{460}}{f_{870}}$$

Solving for the reactance at a frequency of 870 Hz gives

$$X_{C, 870} = X_{C, 460} \frac{f_{460}}{f_{870}} = (68 \Omega) \left(\frac{460 \text{ Hz}}{870 \text{ Hz}}\right) = 36 \Omega$$

4. **REASONING** The rms voltage $V_{\text{rms}}$ and current $I_{\text{rms}}$ in a capacitor are related according to $V_{\text{rms}} = I_{\text{rms}} X_C$ (Equation 23.1). $X_C$ is the capacitive reactance $X_C = \frac{1}{2\pi f C}$ (Equation 23.2), where $f$ is the frequency in hertz (Hz) and $C$ is the capacitance of the capacitor. We will apply these equations to the capacitor when connected to each of the
generators. Although we are not given a value for the capacitance, we will see that it is eliminated algebraically from the calculation.

**SOLUTION** Substituting Equation 23.2 for the capacitive reactance into Equation 23.1, we obtain

\[ V_{\text{rms}} = I_{\text{rms}}X_C = I_{\text{rms}} \left( \frac{1}{2\pi f C} \right) \]  

(1)

Applying Equation (1) to the capacitor when connected to generator 1 and generator 2, we have

\[ (V_{\text{rms}})_1 = (I_{\text{rms}})_1 \left( \frac{1}{2\pi f_1 C} \right) \quad \text{and} \quad (V_{\text{rms}})_2 = (I_{\text{rms}})_2 \left( \frac{1}{2\pi f_2 C} \right) \]

Dividing the equation on the right by the equation on the left, we note that the capacitance \( C \) is eliminated algebraically and find that

\[ \frac{(V_{\text{rms}})_2}{(V_{\text{rms}})_1} = \frac{(I_{\text{rms}})_2 / (2\pi f_2 C)}{(I_{\text{rms}})_1 / (2\pi f_1 C)} = \frac{(I_{\text{rms}})_2 (2\pi f_1 C)}{(2\pi f_2 C)(I_{\text{rms}})_1} = \frac{(I_{\text{rms}})_2 f_1}{f_2 (I_{\text{rms}})_1} \]

(2)

Solving Equation (2) for the voltage of the second generator gives

\[ (V_{\text{rms}})_2 = \frac{(V_{\text{rms}})_1 (I_{\text{rms}})_2 f_1}{f_2 (I_{\text{rms}})_1} = \frac{(2.0 \text{ V})(85 \times 10^{-3} \text{ A})(3.4 \times 10^3 \text{ Hz})}{(5.0 \times 10^3 \text{ Hz})(35 \times 10^{-3} \text{ A})} = 3.3 \text{ V} \]

5. **SSM REASONING** The rms current in a capacitor is \( I_{\text{rms}} = V_{\text{rms}}/X_C \), according to Equation 23.1. The capacitive reactance is \( X_C = 1/(2\pi f C) \), according to Equation 23.2. For the first capacitor, we use \( C = C_1 \) in these expressions. For the two capacitors in parallel, we use \( C = C_p \), where \( C_p \) is the equivalent capacitance from Equation 20.18 (\( C_p = C_1 + C_2 \)). Taking the difference between the currents and using the given data, we can obtain the desired value for \( C_2 \). The capacitance \( C_1 \) is unknown, but it will be eliminated algebraically from the calculation.

**SOLUTION** Using Equations 23.1 and 23.2, we find that the current in a capacitor is

\[ I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{V_{\text{rms}}}{1 / (2\pi f C)} = V_{\text{rms}} 2\pi f C \]

Applying this result to the first capacitor and the parallel combination of the two capacitors, we obtain

\[ I_1 = V_{\text{rms}} 2\pi f C_1 \quad \text{and} \quad I_{\text{Combination}} = V_{\text{rms}} 2\pi f (C_1 + C_2) \]

Subtracting \( I_1 \) from \( I_{\text{Combination}} \) reveals that
\[ I_{\text{Combination}} - I_1 = V_{\text{rms}} 2\pi f C_1 + C_2 \hbar V_{\text{rms}} 2\pi f C_1 = V_{\text{rms}} 2\pi f C_2 \]

Solving for \( C_2 \) gives
\[
C_2 = \frac{I_{\text{Combination}} - I_1}{V_{\text{rms}} 2\pi f} = \frac{0.18 \text{ A}}{\frac{24 \text{ V}}{20 \text{ Hz}}} = 2.7 \times 10^{-6} \text{ F}
\]

6. **REASONING** The rms current in the circuit is given by \( I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} \) (Equation 23.1), where \( V_{\text{rms}} \) is the rms voltage of the generator, and \( X_C = 1/(2\pi f C) \) is the capacitive reactance (see Equation 23.2). For a given voltage, smaller reactances lead to greater currents.

When two capacitors are connected in parallel, the equivalent capacitance \( C_p \) is given by \( C_p = C_1 + C_2 \) (Equation 20.18), where \( C_1 \) and \( C_2 \) are the individual capacitances. Therefore, \( C_p \) is greater than either \( C_1 \) or \( C_2 \). Thus, when the capacitors are connected in parallel, the greater capacitance leads to a smaller reactance (\( C \) is in the denominator in Equation 23.2), which in turn leads to a greater current. As a result, the current delivered by the generator increases when the second capacitor is connected in parallel with the first capacitor.

The capacitance of a parallel plate capacitor is given by \( C = \kappa \varepsilon_0 A/d \) (Equation 19.10), where \( \kappa \) is the dielectric constant of the material between the plates, \( \varepsilon_0 \) is the permittivity of free space, \( A \) is the area of each plate, and \( d \) is the separation between the plates. When the capacitor is empty, \( \kappa = 1 \), so that \( C = \kappa C_{\text{empty}} \). Thus, the capacitance increases when the dielectric material is inserted.

**SOLUTION** Using Equation 23.1 to express the current as \( I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} \) and Equation 23.2 to express the capacitive reactance as \( X_C = 1/(2\pi f C) \), we have for the current that
\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{V_{\text{rms}}}{1/2\pi f C} = V_{\text{rms}} 2\pi f C
\]

Applying this result to the case where the empty capacitor \( C_1 \) is connected alone to the generator and to the case where the “full” capacitor \( C_2 \) (which contains the dielectric material) is connected in parallel with \( C_1 \), we obtain
\[
\begin{align*}
I_{1,\text{rms}} &= \frac{V_{\text{rms}} 2\pi f C_1}{C_1} \quad \text{and} \quad I_{p,\text{rms}} = \frac{V_{\text{rms}} 2\pi f C_p}{C_p} = \frac{V_{\text{rms}} 2\pi f C_p}{C_1}
\end{align*}
\]

Dividing the two expressions gives
\[
\frac{I_{p,\text{rms}}}{I_{1,\text{rms}}} = \frac{V_{\text{rms}} 2\pi f C_p}{V_{\text{rms}} 2\pi f C_1} = \frac{C_p}{C_1}
\]
According to Equation 20.18, the equivalent capacitance of the two capacitors in parallel is 
\[ C_p = C_1 + C_2, \]
so that the result for the current ratio becomes
\[ \frac{I_{p,\text{rms}}}{I_{1,\text{rms}}} = \frac{C_1 + C_2}{C_1} = 1 + \frac{C_2}{C_1}. \]

Since the capacitance of a filled capacitor is given by Equation 19.10 as 
\[ C = \kappa \varepsilon_0 A/d, \]
we find that
\[ \frac{I_{p,\text{rms}}}{I_{1,\text{rms}}} = 1 + \frac{\kappa \varepsilon_0 A/d}{\varepsilon_0 A/d} = 1 + \kappa. \]

Solving for \( I_{p,\text{rms}} \) gives
\[ I_{p,\text{rms}} = I_{1,\text{rms}} (1 + \kappa) = 0.22 \text{ A} + 4.2 \text{ A} = 1.1 \text{ A} \]

7. **REASONING** The capacitance \( C \) is related to the capacitive reactance \( X_C \) and the frequency \( f \) via Equation 23.2 as 
\[ C = \frac{1}{2 \pi f X_C}. \]
The capacitive reactance, in turn is related to the rms-voltage \( V_{\text{rms}} \) and the rms-current \( I_{\text{rms}} \) by \( X_C = V_{\text{rms}}/I_{\text{rms}} \) (see Equation 23.1). Thus, the capacitance can be written as 
\[ C = \frac{I_{\text{rms}}}{2 \pi f V_{\text{rms}}}. \]
The magnitude of the maximum charge \( q \) on one plate of the capacitor is, from Equation 19.8, the product of the capacitance \( C \) and the peak voltage \( V \).

**SOLUTION**

a. Recall that the rms-voltage \( V_{\text{rms}} \) is related to the peak voltage \( V \) by \( V_{\text{rms}} = \frac{V}{\sqrt{2}} \). The capacitance is, then,
\[ C = \frac{I_{\text{rms}}}{2 \pi f V_{\text{rms}}} = \frac{3.0 \text{ A}}{2 \pi f 50 \text{ Hz} \cdot \frac{40 \text{ V}}{\sqrt{2}}} = 6.4 \times 10^{-6} \text{ F} \]

b. The maximum charge on one plate of the capacitor is
\[ q = CV = 6.4 \times 10^{-6} \text{ F} \cdot 40 \text{ V} = 9.0 \times 10^{-4} \text{ C} \]

8. **REASONING AND SOLUTION** Equations 23.1 and 23.2 indicate that the rms current in a capacitor is \( I = \frac{V}{X_C} \), where \( V \) is the rms voltage and \( X_C = \frac{1}{2 \pi f C} \). Therefore, the current is \( I = V \frac{2 \pi f}{C} \). For a single capacitor \( C = C_1 \), and we have
\[ I = V \frac{2 \pi f}{C_1} \]

For two capacitors in series, Equation 20.19 indicates that the equivalent capacitance can be
obtained from \( C^{-1} = C_1^{-1} + C_2^{-1} \), which can be solved to show that \( C = C_1 C_2 / (C_1 + C_2) \). The total series current is, then,

\[
I_{\text{series}} = V 2\pi f C = V 2\pi f \frac{C_1 C_2}{C_1 + C_2}
\]

The series current is one-third of the current \( I \). It follows, therefore, that

\[
\frac{I_{\text{series}}}{I} = \frac{V 2\pi f}{V 2\pi f C_1} \frac{C_1 + C_2}{C_1} = \frac{C_2}{C_1 + C_2} = \frac{1}{3} \quad \text{or} \quad \frac{C_1}{C_2} = 2
\]

For two capacitors in parallel, Equation 20.18 indicates that the equivalent capacitance is \( C = C_1 + C_2 \). The total current in this case is

\[
I_{\text{parallel}} = V 2\pi f C = V 2\pi f \frac{C_1 + C_2}{C_1 + C_2} I
\]

The ratio of \( I_{\text{parallel}} \) to the current \( I \) in the single capacitor is

\[
\frac{I_{\text{parallel}}}{I} = \frac{V 2\pi f}{V 2\pi f C_1} \frac{C_1 + C_2}{C_1} = \frac{C_1 + C_2}{C_1} = 1 + \frac{C_2}{C_1} = 1 + \frac{1}{2} = \frac{3}{2}
\]

9. **SSM REASONING** The rms current can be calculated from Equation 23.3, \( I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} \), provided that the inductive reactance is obtained first. Then the peak value of the current \( I_0 \) supplied by the generator can be calculated from the rms current \( I_{\text{rms}} \) by using Equation 20.12, \( I_0 = \sqrt{2} I_{\text{rms}} \).

**SOLUTION** At the frequency of \( f = 620 \text{ Hz} \), we find, using Equations 23.4 and 23.3, that

\[
X_L = 2\pi f L = 2\pi (620 \text{ Hz})(8.2 \times 10^{-3} \text{ H}) = 32 \Omega
\]

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{10.0 \text{ V}}{32 \Omega} = 0.31 \text{ A}
\]

Therefore, from Equation 20.12, we find that the peak value \( I_0 \) of the current supplied by the generator must be

\[
I_0 = \sqrt{2} I_{\text{rms}} = \sqrt{2} 0.31 \text{ A} = 0.44 \text{ A}
\]

10. **REASONING** The rms voltage \( V_{\text{rms}} \) and current \( I_{\text{rms}} \) in an inductor are related according to \( V_{\text{rms}} = I_{\text{rms}} X_L \) (Equation 23.3). \( X_L \) is the capacitive reactance \( X_L = 2\pi f L \) (Equation 23.4), where \( f \) is the frequency in hertz (Hz) and \( L \) is the inductance of the inductor. Since we have values for \( V_{\text{rms}}, f, \) and \( L \), we can use these equations to calculate the unknown current \( I_{\text{rms}} \).
**SOLUTION** Substituting Equation 23.4 for the inductive reactance into Equation 23.3, we obtain

$$V_{\text{rms}} = I_{\text{rms}}X_L = I_{\text{rms}}(2\pi f L)$$

(1)

Solving Equation (1) for $I_{\text{rms}}$, we find that

$$V_{\text{rms}} = I_{\text{rms}}(2\pi f L)$$  \[\text{or}\]  $$I_{\text{rms}} = \frac{V_{\text{rms}}}{2\pi f L} = \frac{55 \text{ V}}{2\pi(650 \text{ Hz})(0.080 \text{ H})} = 0.17 \text{ A}$$

11. **REASONING** The rms voltage $V_{\text{rms}}$ across the inductor is given by $V_{\text{rms}} = I_{\text{rms}}X_L$ (Equation 23.3), where $I_{\text{rms}}$ is the rms current in the circuit, and $X_L$ is the inductive reactance. The inductor is the only circuit element connected to the generator, so the rms voltage across the inductor is equal to the rms generator voltage: $V_{\text{rms}} = 15.0 \text{ V}$.

**SOLUTION** Solving Equation 23.3 for $X_L$, we obtain

$$X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{15.0 \text{ V}}{0.610 \text{ A}} = 24.6 \Omega$$

12. **REASONING** The inductance $L$ of the inductor determines its inductive reactance $X_L$ according to $X_L = 2\pi fL$ (Equation 23.4), where $f$ is the frequency of the generator. When the inductor is connected to the terminals of the generator, the rms voltage $V_{\text{rms}}$ of the generator drives an rms current $I_{\text{rms}}$ that depends upon the inductive reactance via $X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}}$ (Equation 23.3). We note that the generator frequency is given in kHz, where $1 \text{ kHz} = 10^3 \text{ Hz}$, and the rms current is given in mA, where $1 \text{ mA} = 10^{-3} \text{ A}$.

**SOLUTION** Solving $X_L = 2\pi fL$ (Equation 23.4) for $L$ yields

$$L = \frac{X_L}{2\pi f}$$

(2)

Substituting $X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}}$ (Equation 23.3) into Equation (1), we find that

$$L = \frac{X_L}{2\pi f} = \frac{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)}{2\pi f} = \frac{V_{\text{rms}}}{2\pi f I_{\text{rms}}} = \frac{39 \text{ V}}{2\pi(7.5\times10^3 \text{ Hz})(42\times10^{-3} \text{ A})} = 0.020 \text{ H}$$
13. **REASONING** Since the capacitor and the inductor are connected in parallel, the voltage across each of these elements is the same or $V_L = V_C$. Using Equations 23.3 and 23.1, respectively, this becomes $I_{\text{rms}} X_L = I_{\text{rms}} X_C$. Since the currents in the inductor and capacitor are equal, this relation simplifies to $X_L = X_C$. Therefore, we can find the value of the inductance by equating the expressions (Equations 23.4 and 23.2) for the inductive reactance and the capacitive reactance, and solving for $L$.

**SOLUTION** Since $X_L = X_C$, we have

$$2\pi f L = \frac{1}{2\pi f C}$$

Therefore, the value of the inductance is

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (60.0 \text{ Hz})^2 (40.0 \times 10^{-6} \text{ F})} = 0.176 \text{ H} = 176 \text{ mH}$$

14. **REASONING** The current in $L_1$ is given by Equation 23.3 as $I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L}$, where $X_L = 2\pi f L_1$ (Equation 23.4) is the inductive reactance of $L_1$. This current does not depend in any way on $L_2$ and exists whether or not $L_2$ is present.

The current delivered to the parallel combination is the sum of the currents delivered to each inductance and is, therefore, greater than either individual current. The current in $L_2$ is given by $I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L}$ (Equation 23.3), where $X_L = 2\pi f L_2$ is the inductive reactance of $L_2$ according to Equation 23.4. This current does not depend in any way on $L_1$ and exists whether or not $L_1$ is present.

**SOLUTION** Using Equation 23.3 to express the current as $I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L}$ and Equation 23.4 to express the inductive reactance as $X_L = 2\pi f L$, we have for the current that

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{V_{\text{rms}}}{2\pi f L}$$

Applying this result to the case where $L_1$ or $L_2$ is connected alone to the generator, we obtain

$$I_{1,\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{V_{\text{rms}}}{2\pi f L_1} \quad \text{and} \quad I_{2,\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{V_{\text{rms}}}{2\pi f L_2}$$

The current delivered to $L_1$ alone is

$$I_{1,\text{rms}} = \frac{V_{\text{rms}}}{2\pi f L_1} = \frac{240 \text{ V}}{2\pi (200 \text{ Hz}) (40.0 \times 10^{-3} \text{ H})} = 2.9 \text{ A}$$
The current delivered to the parallel combination of $L_1$ and $L_2$ is the sum of that delivered individually to each inductor and is

$$I_{p, rms} = I_{1, rms} + I_{2, rms} = \frac{V_{rms}}{2\pi f L_1} + \frac{V_{rms}}{2\pi f L_2} = \frac{V_{rms}}{2\pi f L_1} + \frac{1}{L_2}$$

$$= \frac{240 \text{ V}}{2\pi \cdot 600 \text{ Hz}} \left( \frac{1}{6.0 \times 10^{-3} \text{ H}} + \frac{1}{9.0 \times 10^{-3} \text{ H}} \right) = 4.8 \text{ A}$$

15. **SSM REASONING**

   a. The inductive reactance $X_L$ depends on the frequency $f$ of the current and the inductance $L$ through the relation $X_L = 2\pi fL$ (Equation 23.4). This equation can be used directly to find the frequency of the current.

   b. The capacitive reactance $X_C$ depends on the frequency $f$ of the current and the capacitance $C$ through the relation $X_C = 1/(2\pi fC)$ (Equation 23.2). By setting $X_C = X_L$ as specified in the problem statement, the capacitance can be found.

   c. Since the inductive reactance is directly proportional to the frequency, tripling the frequency triples the inductive reactance.

   d. The capacitive reactance is inversely proportional to the frequency, so tripling the frequency reduces the capacitive reactance by a factor of one-third.

**SOLUTION**

a. The frequency of the current is

$$f = \frac{X_L}{2\pi L} = \frac{2.10 \times 10^3 \Omega}{2\pi \left(30.0 \times 10^{-3} \text{ H}\right)} = 1.11 \times 10^4 \text{ Hz}$$

b. The capacitance is

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \left(1.11 \times 10^4 \text{ Hz}\right) \left(2.10 \times 10^3 \Omega\right)} = 6.83 \times 10^{-9} \text{ F}$$

   c. Since $X_L = 2\pi fL$, tripling the frequency $f$ causes $X_L$ to also triple:

$$X_L = 3 \left(2.10 \times 10^3 \Omega\right) = 6.30 \times 10^3 \Omega$$

   d. Since $X_C = 1/(2\pi fC)$, tripling the frequency $f$ causes $X_C$ to decrease by a factor of 3:

$$X_C = \frac{1}{3} \left(2.10 \times 10^3 \Omega\right) = 7.00 \times 10^2 \Omega$$
16. **REASONING AND SOLUTION** Equations 23.3 and 23.4 indicate that the rms current in a single inductance \( L_1 \) is \( I_1 = \frac{V}{\sqrt{2} \pi f L_1} \), where \( V \) is the rms voltage and \( X_{L1} = 2\pi f L_1 \). Therefore, the current is \( I_1 = \frac{V}{\sqrt{2} \pi f L_1} \). Similarly, the current in the second inductor connected across the terminals of the generator is \( I_2 = \frac{V}{\sqrt{2} \pi f L_2} \). The total current delivered by the generator is the sum of these two values:

\[
I_{\text{total}} = I_1 + I_2 = \frac{V}{2\pi f L_1} + \frac{V}{2\pi f L_2}
\]

But this same total current is delivered to the single inductance \( L \), so it also follows that \( I_{\text{total}} = \frac{V}{\sqrt{2} \pi f L} \). Equating the two expressions for \( I_{\text{total}} \) shows that

\[
\frac{V}{2\pi f L} = \frac{V}{2\pi f L_1} + \frac{V}{2\pi f L_2} \quad \text{or} \quad \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}
\]

Using this result, we determine the value of \( L \) as follows:

\[
\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or} \quad L = \frac{L_1 L_2}{L_1 + L_2} = \frac{0.030 \text{ H} \cdot 0.060 \text{ H}}{0.030 \text{ H} + 0.060 \text{ H}} = 0.020 \text{ H}
\]

17. **SSM REASONING** The voltage supplied by the generator can be found from Equation 23.6, \( V_{\text{rms}} = I_{\text{rms}} Z \). The value of \( I_{\text{rms}} \) is given in the problem statement, so we must obtain the impedance of the circuit.

**SOLUTION** The impedance of the circuit is, according to Equation 23.7,

\[
Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(275 \Omega)^2 + (648 \Omega - 415 \Omega)^2} = 3.60 \times 10^2 \Omega
\]

The rms voltage of the generator is

\[
V_{\text{rms}} = I_{\text{rms}} Z = (0.233 \text{ A})(3.60 \times 10^2 \Omega) = 83.9 \text{ V}
\]

18. **REASONING** As discussed in Section 23.3, the power factor is

\[
\cos \phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}},
\]

where \( R \) is the resistance, \( X_L \) is the inductive reactance, and \( X_C \) is the capacitive reactance of the circuit. The inductive reactance is given by \( X_L = 2\pi f L \) (Equation 23.4), where \( f \) is the frequency in hertz (Hz) and \( L \) is the inductance. The capacitive reactance is given by \( X_C = \frac{1}{2\pi f C} \) (Equation 23.2), where \( C \) is the capacitance.
**SOLUTION** Using Equation 23.4 and Equation 23.2 to substitute for the reactances, we find that the power factor is

\[
\cos \phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}
\]

\[
= \frac{47.0 \ \Omega}{\sqrt{(47.0 \ \Omega)^2 + \left[2\pi (2550 \text{ Hz})(4.00 \times 10^{-3} \text{ H}) - \frac{1}{2\pi (2550 \text{ Hz})(2.00 \times 10^{-6} \text{ F})}\right]^2}}
\]

\[
= 0.819
\]

19. **SSM REASONING** We can use the equations for a series RCL circuit to solve this problem provided that we set \( X_C = 0 \ \Omega \) since there is no capacitor in the circuit. The current in the circuit can be found from Equation 23.6, \( V_{rms} = I_{rms} Z \), once the impedance of the circuit has been obtained. Equation 23.8, \( \tan \phi = (X_L - X_C)/R \), can then be used (with \( X_C = 0 \ \Omega \)) to find the phase angle between the current and the voltage.

**SOLUTION** The inductive reactance is (Equation 23.4)

\[
X_L = 2\pi f L = 2\pi (106 \text{ Hz})(0.200 \text{ H}) = 133 \ \Omega
\]

The impedance of the circuit is

\[
Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X_L^2} = \sqrt{(215 \ \Omega)^2 + (133 \ \Omega)^2} = 253 \ \Omega
\]

a. The current through each circuit element is, using Equation 23.6,

\[
I_{rms} = \frac{V_{rms}}{Z} = \frac{234 \ \text{V}}{253 \ \Omega} = 0.925 \ \text{A}
\]

b. The phase angle between the current and the voltage is, according to Equation 23.8 (with \( X_C = 0 \ \Omega \)),

\[
\tan \phi = \frac{X_L - X_C}{R} = \frac{X_L}{R} = \frac{133 \ \Omega}{215 \ \Omega} = 0.619 \quad \text{or} \quad \phi = \tan^{-1}(0.619) = 31.8^\circ
\]
20. **REASONING** The phase angle is given by \( \tan \phi = (X_L - X_C)/R \) (Equation 23.8). When a series circuit contains only a resistor and a capacitor, the inductive reactance \( X_L \) is zero, and the phase angle is negative, signifying that the current leads the voltage of the generator. The impedance is given by Equation 23.7 with \( X_L = 0 \Omega \), or \( Z = \sqrt{R^2 + X_C^2} \). Since the phase angle \( \phi \) and the impedance \( Z \) are given, we can use these relations to find the resistance \( R \) and \( X_C \).

**SOLUTION** Since the phase angle is negative, we can conclude that only a resistor and a capacitor are present. Using Equations 23.8, then, we have

\[
\tan \phi = \frac{-X_C}{R} \quad \text{or} \quad X_C = -R \tan \theta \approx 75.0^\circ \approx 3.73 R
\]

According to Equation 23.7, the impedance is

\[
Z = 192 \Omega = \sqrt{R^2 + X_C^2}
\]

Substituting \( X_C = 3.73R \) into this expression for \( Z \) gives

\[
192 \Omega = \sqrt{R^2 + (3.73 R)^2} = \sqrt{14.9 R^2} \quad \text{or} \quad 192 \Omega = 14.9 R^2
\]

\[
R = \sqrt{\frac{192 \Omega}{14.9}} = \frac{49.7 \Omega}{14.9} \approx 3.33 \Omega
\]

Using the fact that \( X_C = 3.73 R \), we obtain

\[
X_C = 3.73 (49.7 \Omega) = 185 \Omega
\]

21. **REASONING** For a series RCL circuit the total impedance \( Z \) and the phase angle \( \phi \) are given by

\[
Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (23.7) \quad \tan \phi = \frac{X_L - X_C}{R} \quad (23.8)
\]

where \( R \) is the resistance, \( X_L \) is the inductive reactance, and \( X_C \) is the capacitive reactance. In the present case, there is no capacitance, so that \( X_C = 0 \Omega \). Therefore, these equations simplify to the following:

\[
Z = \sqrt{R^2 + X_L^2} \quad (1) \quad \tan \phi = \frac{X_L}{R} \quad (2)
\]

We are given neither \( R \) nor \( X_L \). However, we do know the current and voltage when only the resistor is connected and can determine \( R \) from these values using \( R = \frac{V_{\text{rms}}}{I_{\text{rms}}} \) (Equation 20.14). In addition, we know the current and voltage when only the inductor is
connected and can determine \( X_L \) from these values using \( X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} \) (Equation 23.3). Since the generator frequency is fixed, this value for \( X_L \) also applies for the series combination of the resistor and the inductor.

**SOLUTION**  With only the resistor connected, Equation 20.14 indicates that the resistance is

\[
R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{112 \text{ V}}{0.500 \text{ A}} = 224 \text{ \Omega}
\]

With only the inductor connected, Equation 23.3 indicates that the inductive reactance is

\[
X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{112 \text{ V}}{0.400 \text{ A}} = 2.80 \times 10^2 \text{ \Omega}
\]

a. Using these values for \( R \) and \( X_L \) in Equations (1) and (2), we find that the impedance is

\[
Z = \sqrt{R^2 + X_L^2} = \sqrt{(224 \text{ \Omega})^2 + (2.80 \times 10^2 \text{ \Omega})^2} = 359 \text{ \Omega}
\]

b. The phase angle \( \phi \) between the current and the voltage of the generator is

\[
\tan \phi = \frac{X_L}{R} \quad \text{or} \quad \phi = \tan^{-1}\left( \frac{X_L}{R} \right) = \tan^{-1}\left( \frac{2.80 \times 10^2 \text{ \Omega}}{224 \text{ \Omega}} \right) = 51.3^\circ
\]

22. **REASONING**  Since, on the average, only the resistor consumes power, the average power dissipated in the circuit is \( \bar{P} = I_{\text{rms}}^2 R \) (Equation 20.15b), where \( I_{\text{rms}} \) is the rms current and \( R \) is the resistance. The current is related to the voltage \( V \) of the generator and the impedance \( Z \) of the circuit by \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \) (Equation 23.6). Thus, the average power can be written as

\[
\bar{P} = \frac{V^2 R}{Z^2}
\]

The impedance of the circuit is (see Equation 23.7) \( Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X_C^2} \), since there is no inductor in the circuit. Therefore, the expression for the average power becomes

\[
\bar{P} = \frac{V^2 R}{Z^2} = \frac{V^2 R}{R^2 + X_C^2}
\]

**SOLUTION**  The capacitive reactance is

\[
X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi (60.0 \text{ Hz})(1.1 \times 10^{-6} \text{ F})} = 2400 \text{ \Omega}
\]

(23.2)

The average power dissipated is
\[
\bar{P} = \frac{V^2 R}{R^2 + X_C^2} = \frac{(120 \text{ V})^2 (2700 \Omega)}{(2700 \Omega)^2 + (2400 \Omega)^2} = 3.0 \text{ W}
\]

23. **REASONING** From Figure 23.11 we see that \( V_o^2 = 0 + V_R^2 \). Since \( V_L = 0 \) V \((L = 0 \text{ H})\), and we know \( V_0 \) and \( V_R \), we can use this equation to find \( V_C \).

**SOLUTION** Solving the equation above for \( V_C \) gives

\[
V_C = \sqrt{V_o^2 - V_R^2} = \sqrt{\frac{120 \text{ V}}{6} - \frac{24 \text{ V}}{4} \cdot \frac{24 \text{ V}}{4}} = 38 \text{ V}
\]

24. **REASONING** The rms current \( I_{\text{rms}} \) in the circuit in part c of the drawing is equal to the rms voltage \( V_{\text{rms}} \) divided by the impedance \( Z \) or \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \) (Equation 23.6). The impedance \( Z \) of a series RC circuit is given by Equation 23.7 with \( X_L = 0 \) \(\Omega\), since there is no inductance in the circuit; \( Z = \sqrt{R^2 + X_C^2} \). The resistance \( R \) is known and the capacitive reactance \( X_C \) can be obtained from the relation \( X_C = 1/(2\pi f C) \) (Equation 23.2), where \( C \) is the capacitance and \( f \) is the frequency. The capacitance, however, is related to the time constant \( \tau \) of the circuit in part a of the drawing. The time constant of an RC circuit is the time for the capacitor to lose 63.2% of its initial charge (see the discussion in Section 20.13), and it is equal to the product of the resistance \( R \) and the capacitance \( C \); \( \tau = RC \) (Equation 20.21).

**SOLUTION** Substituting \( Z = \sqrt{R^2 + X_C^2} \) into \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \) gives

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_C^2}}
\]

(1)

Substituting \( X_C = 1/(2\pi f C) \) (Equation 23.2) into Equation (1) allows us to write the rms current in the circuit in part c of the drawing as follows:

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_C^2}} = \frac{V_{\text{rms}}}{ \sqrt{R^2 + \left( \frac{1}{2\pi f C} \right)^2} }
\]

Substituting \( C = \tau/R \) (Equation 20.21) into Equation (1), we arrive at an expression for the rms current:
\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\frac{R}{2\pi f \tau}\right)^2}}
\]

\[
= \frac{24 \text{ V}}{\sqrt{(18 \ \Omega)^2 + \left[\frac{18 \Omega}{2\pi (380 \text{ Hz})(3.0 \times 10^{-4} \text{ s})}\right]^2}} = 0.78 \text{ A}
\]

25. **SSM REASONING**  We can use the equations for a series RCL circuit to solve this problem, provided that we set \(X_L = 0\) since there is no inductance in the circuit. Thus, according to Equations 23.6 and 23.7, the current in the circuit is \(I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_C^2}}\). When the frequency \(f\) is very large, the capacitive reactance is zero, or \(X_C = 0\), in which case the current becomes \(I_{\text{rms}} \text{(large } f\text{)} = \frac{V_{\text{rms}}}{R}\). When the current \(I_{\text{rms}}\) in the circuit is one-half the value of \(I_{\text{rms}} \text{(large } f\text{)}\) that exists when the frequency is very large, we have

\[
\frac{I_{\text{rms}}}{I_{\text{rms}} \text{(large } f\text{)}} = \frac{1}{2}
\]

We can use these expressions to write the ratio above in terms of the resistance and the capacitive reactance. Once the capacitive reactance is known, the frequency can be determined.

**SOLUTION**  The ratio of the currents is

\[
\frac{I_{\text{rms}}}{I_{\text{rms}} \text{(large } f\text{)}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_C^2}} \cdot \frac{\sqrt{R^2 + X_C^2}}{V_{\text{rms}} / R} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{1}{2} \quad \text{or} \quad \frac{R^2}{R^2 + X_C^2} = \frac{1}{4}
\]

Taking the reciprocal of this result gives

\[
\frac{R^2 + X_C^2}{R^2} = 4 \quad \text{or} \quad 1 + \frac{X_C^2}{R^2} = 4
\]

Therefore,

\[
\frac{X_C}{R} = \sqrt{3}
\]

According to Equation 23.2, \(X_C = \frac{1}{2\pi f C}\), so it follows that

\[
\frac{X_C}{R} = \frac{1}{2\pi f C} \frac{g}{R} = \sqrt{3}
\]

Thus, we find that
\[
f = \frac{1}{\pi RC\sqrt{3}} = \frac{1}{2 \pi \times 3 \times 0.0 \times 10^{-6} \times \sqrt{3}} = 270 \text{ Hz}
\]

26. **REASONING** For a series RCL circuit the total impedance \( Z \) and the phase angle \( \phi \) are given by

\[
Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (23.7)
\]

\[
\tan \phi = \frac{X_L - X_C}{R} \quad (23.8)
\]

where \( R \) is the resistance, \( X_L \) is the inductive reactance, and \( X_C \) is the capacitive reactance. In the present case, there is no inductance, so \( X_L = 0 \Omega \). Therefore, these equations simplify to the following:

\[
Z = \sqrt{R^2 + X_C^2} \quad (1)
\]

\[
\tan \phi = -\frac{X_C}{R} \quad (2)
\]

Values are given for \( Z \) and \( \phi \), so we can solve for \( R \) and \( X_C \).

**SOLUTION** Using the value given for the phase angle in Equation (2), we find that

\[
\tan \phi = \tan (-9.80^\circ) = -0.173 = -\frac{X_C}{R} \quad \text{or} \quad X_C = 0.173R \quad (3)
\]

Substituting this result for \( X_C \) into Equation (1) gives

\[
Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (0.173R)^2} = R \sqrt{1 + (0.173)^2}
\]

Solving for \( R \) reveals that

\[
R = \frac{Z}{\sqrt{1 + (0.173)^2}} = \frac{4.50 \times 10^2 \Omega}{\sqrt{1 + (0.173)^2}} = 443 \Omega
\]

Using this value for \( R \) in Equation (3), we find that

\[
X_C = 0.173R = 0.173(443 \Omega) = 76.6 \Omega
\]

27. **REASONING** The instantaneous value of the generator voltage is \( V(t) = V_0 \sin 2\pi ft \), where \( V_0 \) is the peak voltage and \( f \) is the frequency. We will see that the inductive reactance is greater than the capacitive reactance, \( X_L > X_C \), so that the current in the circuit lags the voltage by \( \pi/2 \) radians, or \( 90^\circ \). Thus, the current in the circuit obeys the relation \( I(t) = I_0 \sin (2\pi ft - \pi/2) \), where \( I_0 \) is the peak current.

**SOLUTION**

a. The instantaneous value of the voltage at a time of \( 1.20 \times 10^{-4} \) s is
\[ V(t) = V_0 \sin 2\pi ft = (32.0 \text{ V}) \sin \left[ 2\pi \left( 1.50 \times 10^3 \text{ Hz} \right) \left( 1.20 \times 10^{-4} \text{ s} \right) \right] = 29.0 \text{ V} \]

Note: When evaluating the sine function in the expression above, be sure to set your calculator to the \textit{radian} mode.

b. The inductive and capacitive reactances are

\[ X_L = 2\pi fL = 2\pi \left( 1.50 \times 10^3 \text{ Hz} \right) \left( 7.20 \times 10^{-3} \text{ H} \right) = 67.9 \Omega \quad (23.4) \]

\[ X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \left( 1.50 \times 10^3 \text{ Hz} \right) \left( 6.60 \times 10^{-6} \text{ F} \right)} = 16.1 \Omega \quad (23.2) \]

Since \( X_L \) is greater than \( X_C \), the current lags the voltage by \( \pi/2 \) radians. Thus, the instantaneous current in the circuit is \( I(t) = I_0 \sin (2\pi ft - \pi/2) \), where \( I_0 = V_0/Z \). The impedance \( Z \) of the circuit is

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(0 \Omega)^2 + (67.9 \Omega - 16.1 \Omega)^2} = 51.8 \Omega \quad (23.7) \]

The instantaneous current is

\[ I = \frac{V_0}{Z} \sin \left( 2\pi ft - \frac{1}{2} \pi \right) \]

\[ = \left( \frac{32.0 \text{ V}}{51.8 \Omega} \right) \sin \left[ 2\pi \left( 1.50 \times 10^3 \text{ Hz} \right) \left( 1.20 \times 10^{-4} \text{ s} \right) - \frac{1}{2} \pi \right] = -0.263 \text{ A} \]

28. \textit{REASONING} The rms voltages across the inductor \( L \) and the capacitor \( C \) are given, respectively, by \( V_{L,\text{rms}} = I_{\text{rms}}X_L \) (Equation 23.3) and \( V_{C,\text{rms}} = I_{\text{rms}}X_C \) (Equation 23.1), where \( I_{\text{rms}} \) is the rms current in the circuit, \( X_L = 2\pi fL \) (Equation 23.4) is the inductive reactance of the inductor, and \( X_C = \frac{1}{2\pi fC} \) (Equation 23.2) is the capacitive reactance of the capacitor. We know the frequency \( f \) of the generator, but we are not given the rms current in the circuit. We will make use of Equations 23.3 and 23.1 to eliminate the unknown current \( I_{\text{rms}} \), and then solve for the rms voltage across the inductor.

\textit{SOLUTION} Solving Equation 23.1 for \( I_{\text{rms}} \) gives \( I_{\text{rms}} = \frac{V_{C,\text{rms}}}{X_C} \). Substituting this relation into Equation 23.3, we obtain

\[ V_{L,\text{rms}} = I_{\text{rms}}X_L = \frac{V_{C,\text{rms}}X_L}{X_C} \quad (1) \]
Substituting Equations 23.2 and 23.4 for the reactances into Equation (1) yields

\[ V_{L, \text{rms}} = \frac{V_{C, \text{rms}} X_L}{X_C} = \frac{V_{C, \text{rms}} 2\pi f L}{\left( \frac{1}{2\pi f C} \right)} = V_{C, \text{rms}} (2\pi f)^2 L C \]

Therefore, when the rms voltage across the capacitor is \( V_{C, \text{rms}} = 2.20 \, \text{V} \), the rms voltage across the inductor is

\[ V_{L, \text{rms}} = V_{C, \text{rms}} (2\pi f)^2 L C \]

\[ V_{L, \text{rms}} = (2.20 \, \text{V}) \left( 2\pi \times 375 \, \text{Hz} \right)^2 \left( 84.0 \times 10^{-3} \, \text{H} \right) \left( 5.80 \times 10^{-6} \, \text{F} \right) = 5.95 \, \text{V} \]

29. **REASONING** A resistance \( R \) and an inductance \( L \) are connected in series to the generator, but there is no capacitance in the circuit. Therefore, the impedance \( Z \) of the circuit is given by \( Z = \sqrt{R^2 + (X_L - X_C)^2} \) (Equation 23.7), where \( X_L \) is the inductive reactance of the inductor, and the capacitive reactance \( X_C \) is zero:

\[ \sqrt{R^2 + (X_L - 0)^2} = Z \quad \text{or} \quad \sqrt{R^2 + X_L^2} = Z \quad (1) \]

The impedance \( Z \) is related to the rms current \( I_{\text{rms}} \) and the rms generator voltage \( V_{\text{rms}} \) by \( V_{\text{rms}} = I_{\text{rms}} Z \) (Equation 23.6). The rms voltage \( V_{L, \text{rms}} \) across the inductor is given by \( I_{\text{rms}} = \frac{V_{L, \text{rms}}}{X_L} \) (Equation 23.3). We will determine the inductive reactance from the generator frequency \( f \) and the inductance by means of \( X_L = 2\pi f L \) (Equation 23.4).

**SOLUTION** Solving \( V_{\text{rms}} = I_{\text{rms}} Z \) (Equation 23.6) for \( Z \), we obtain

\[ Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} \quad (2) \]

Substituting Equation (2) into Equation (1) yields

\[ \sqrt{R^2 + X_L^2} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \]

(3)

Substituting \( I_{\text{rms}} = \frac{V_{L, \text{rms}}}{X_L} \) (Equation 23.3) into Equation (3), we obtain

\[ \sqrt{R^2 + X_L^2} = \frac{V_{\text{rms}}}{\left( \frac{V_{L, \text{rms}}}{X_L} \right)} \quad \text{or} \quad \sqrt{R^2 + X_L^2} = \frac{V_{\text{rms}} X_L}{V_{L, \text{rms}}} \quad (4) \]
Squaring both sides of Equation (4) and solving for \( R^2 \), we find that

\[
R^2 + X_L^2 = \left( \frac{V_{\text{rms}} X_L}{V_{L,\text{rms}}} \right)^2 \quad \text{or} \quad R^2 = \left( \frac{V_{\text{rms}} X_L}{V_{L,\text{rms}}} \right)^2 - X_L^2 \quad \text{or} \quad R^2 = X_L^2 \left( \frac{V_{\text{rms}}^2}{V_{L,\text{rms}}^2} - 1 \right) \quad (5)
\]

Taking the square root of both sides of Equation (5) yields

\[
R = X_L \sqrt{\frac{V_{\text{rms}}^2}{V_{L,\text{rms}}^2} - 1}
\quad (6)
\]

Substituting \( X_L = 2\pi f L \) (Equation 23.4) into Equation (6), we obtain the resistance \( R \):

\[
R = 2\pi f L \sqrt{\frac{V_{\text{rms}}^2}{V_{L,\text{rms}}^2} - 1} = 2\pi (130 \text{ Hz}) (0.032 \text{ H}) \sqrt{\frac{(8.0 \text{ V})^2}{(2.6 \text{ V})^2} - 1} = 76 \text{ } \Omega
\]

30. **REASONING** The inductance \( L \) and the capacitance \( C \) of a series RCL circuit determine the resonant frequency \( f_0 \) according to

\[
f_0 = \frac{1}{2\pi \sqrt{LC}} \quad (23.10)
\]

As we see from Equation 23.10, the smaller the inductance \( L \), the larger the resonant frequency, and the larger the inductance, the smaller the resonant frequency. Therefore, in part (a) we will use the largest frequency to determine the minimum inductance, and in part (b) we will use the smallest frequency to find the maximum inductance. We note that 1 MHz = 1\times10^6 \text{ Hz}.

**SOLUTION**

a. Squaring both sides of Equation 23.10 and solving for \( L \), we obtain

\[
f_0^2 = \frac{1}{4\pi^2 LC} \quad \text{or} \quad L = \frac{1}{4\pi^2 f_0^2 C} \quad (1)
\]

The minimum inductance is obtained when the resonant frequency is greatest, so Equation (1) gives

\[
L = \frac{1}{4\pi^2 (9.0\times10^6 \text{ Hz})^2 (1.8\times10^{-11} \text{ F})} = 1.7\times10^{-5} \text{ H}
\]

b. Using Equation (1) once more, this time with the smallest resonant frequency, yields the maximum inductance:

\[
L = \frac{1}{4\pi^2 (4.0\times10^6 \text{ Hz})^2 (1.8\times10^{-11} \text{ F})} = 8.8\times10^{-5} \text{ H}
\]
31. **SSM REASONING** The resonant frequency \( f_0 \) of a series RCL circuit depends on the inductance \( L \) and capacitance \( C \) through the relation \( f_0 = \frac{1}{2\pi\sqrt{LC}} \) (Equation 23.10). Since all the variables are known except \( L \), we can use this relation to find the inductance.

**SOLUTION** Solving Equation 23.10 for the inductance gives

\[
L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 \left(690 \times 10^3 \text{ Hz}\right)^2 \left(2.0 \times 10^{-9} \text{ F}\right)} = 2.7 \times 10^{-5} \text{ H}
\]

32. **REASONING** The average power \( \overline{P} \) dissipated in the circuit is \( \overline{P} = I_{\text{rms}} V_{\text{rms}} \cos \phi \) (Equation 23.9), where \( V_{\text{rms}} \) is rms voltage of the generator, \( I_{\text{rms}} \) is the rms current in the circuit, and \( \cos \phi \) is the power factor. The angle \( \phi \) is the angle between the current and voltage phasors and can be determined from \( \tan \phi = \frac{X_L - X_C}{R} \) (Equation 23.8), where \( X_L \) is the inductive reactance, \( X_C \) is the capacitive reactance, and \( R \) is the resistance of the circuit. In Equation 23.9, we have a value for the average power \( \overline{P} \), the current \( I_{\text{rms}} \) and we can obtain a value for the angle \( \phi \) from Equation 23.8 and the fact that the circuit is at resonance. Therefore, Equation 23.9 can be used to determine the voltage \( V_{\text{rms}} \).

**SOLUTION** Solving Equation 23.9 for the voltage, we have

\[
\overline{P} = I_{\text{rms}} V_{\text{rms}} \cos \phi \quad \text{or} \quad V_{\text{rms}} = \frac{\overline{P}}{I_{\text{rms}} \cos \phi}
\] (1)

At resonance in a series RCL circuit we know that \( X_L = X_C \). Therefore, Equation 23.8 becomes

\[
\tan \phi = \frac{X_L - X_C}{R} = \frac{0.00}{R} = 0.00 \quad \text{or} \quad \phi = \tan^{-1}(0.00) = 0.00^\circ
\]

With this value for \( \phi \) Equation (1) reveals that

\[
V_{\text{rms}} = \frac{\overline{P}}{I_{\text{rms}} \cos \phi} = \frac{65.0 \text{ W}}{(0.530 \text{ A}) \cos 0.00^\circ} = 123 \text{ V}
\]

33. **SSM REASONING** The current in an RCL circuit is given by Equation 23.6,

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}
\]

where the impedance \( Z \) of the circuit is given by Equation 23.7 as

\[
Z = \sqrt{R^2 + (X_L - X_C)^2}
\]

The current is a maximum when the impedance is a minimum for a given generator voltage. The minimum impedance occurs when the frequency is \( f_0 \), corresponding to the condition that \( X_L = X_C \), or \( 2\pi f_0 L = 1/(2\pi f_0 C) \). Solving for the frequency \( f_0 \), called the resonant frequency, we find that
\[ f_0 = \frac{1}{2\pi\sqrt{LC}} \]

Note that the resonant frequency depends on the inductance and the capacitance, but does not depend on the resistance.

**SOLUTION**

a. The frequency at which the current is a maximum is

\[ f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(17.0 \times 10^{-3} \text{ H})(12.0 \times 10^{-6} \text{ F})}} = 352 \text{ Hz} \]

b. The maximum value of the current occurs when \( f = f_0 \). This occurs when \( X_L = X_C \), so that \( Z = R \). Therefore, according to Equation 23.6, we have

\[ I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} = \frac{155 \text{ V}}{10.0 \text{ } \Omega} = 15.5 \text{ A} \]

34. **REASONING** The resonant frequency is given by Equation 23.10 as \( f_0 = \frac{1}{d\pi\sqrt{LC}} \) and is inversely proportional to the square root of the circuit capacitance \( C \). Therefore, to reduce the resonant frequency, it is necessary to increase the circuit capacitance. The equivalent capacitance \( C_p \) of two capacitors in parallel is \( C_p = C_1 + C_2 \) (Equation 20.18), which is greater than either capacitance individually. Therefore, to increase the circuit capacitance, \( C_2 \) should be added in parallel with \( C_1 \).

**SOLUTION** The initial resonant frequency is \( f_{01} \). The resonant frequency that results after \( C_2 \) is added in parallel with \( C_1 \) is \( f_{0p} \). Using Equation 23.10, we can express both of these frequencies as follows:

\[ f_{01} = \frac{1}{2\pi\sqrt{LC_1}} \quad \text{and} \quad f_{0p} = \frac{1}{2\pi\sqrt{LC_p}} \]

Here, \( C_p \) is the equivalent parallel capacitance. Dividing the expression for \( f_{01} \) by the expression for \( f_{0p} \) yields

\[ \frac{f_{01}}{f_{0p}} = \frac{1}{d\pi\sqrt{LC_1}} \cdot \frac{d\pi\sqrt{LC_p}}{1} = \frac{C_p}{\sqrt{C_1}} \]

According to Equation 20.18, the equivalent capacitance is \( C_p = C_1 + C_2 \), so that this frequency ratio becomes

\[ \frac{f_{01}}{f_{0p}} = \sqrt{\frac{C_1 + C_2}{C_1}} = \sqrt{1 + \frac{C_2}{C_1}} \]
Squaring both sides of this result and solving for \( C_2 \), we find

\[
\left( \frac{f_{01}}{f_{0p}} \right)^2 = 1 + \frac{C_2}{C_1}
\]

\[
C_2 = C_1 \left[ \left( \frac{f_{01}}{f_{0p}} \right)^2 - 1 \right] = (2.60 \, \mu F) \left[ \left( \frac{7.30 \, \text{kHz}}{5.60 \, \text{kHz}} \right)^2 - 1 \right] = 1.8 \, \mu F
\]

35. **REASONING** As discussed in Section 23.4, a RCL series circuit is at resonance when the current is a maximum and the impedance \( Z \) of the circuit is a minimum. This happens when the inductive reactance \( X_L \) equals the capacitive reactance \( X_C \). When \( X_L = X_C \), the impedance of the circuit (see Equation 23.7) becomes \( Z = \sqrt{R^2 + (X_L - X_C)^2} = R \), so the impedance is due solely to the resistance \( R \). The average power dissipated in the circuit is \( \bar{P} = \frac{V_{\text{rms}}^2}{R} \) (Equation 20.15c). This relation can be used to find the power when the variable resistor is set to another value.

**SOLUTION** The average power \( \bar{P}_1 \) dissipated when the resistance is \( R_1 = 175 \, \Omega \) is

\[
\bar{P}_1 = \frac{V_{\text{rms}}^2}{R_1}
\]

Likewise, the average power \( \bar{P}_2 \) dissipated when the resistance is \( R_2 = 562 \, \Omega \) is

\[
\bar{P}_2 = \frac{V_{\text{rms}}^2}{R_2}
\]

Solving the first equation for \( V_{\text{rms}}^2 \) and substituting the result into the second equation gives

\[
\bar{P}_2 = \frac{V_{\text{rms}}^2}{R_2} = \frac{\bar{P}_1 R_1}{R_2} = \frac{(2.6 \, \text{W}) (175 \, \Omega)}{562 \, \Omega} = 0.81 \, \text{W}
\]

36. **REASONING** The resonant frequency of an RCL circuit is given by \( f_0 = \frac{1}{2\pi \sqrt{LC}} \) (Equation 23.10), where \( L \) is inductance and \( C \) is the capacitance. Because only the inductance of this circuit changes, from \( L_1 = 7.0 \, \text{mH} \) to \( L_2 = 1.5 \, \text{mH} \), we obtain the initial and final resonant frequencies from Equation 23.10:

\[
f_{01} = \frac{1}{2\pi \sqrt{L_1 C}} \quad \text{and} \quad f_{02} = \frac{1}{2\pi \sqrt{L_2 C}}
\]  \hspace{1cm} (1)

We will solve the first of Equations (1) for the capacitance \( C \), and substitute the result into the second of Equations (1).

**SOLUTION** Squaring both sides of the first of Equations (1) and solving for \( C \), we obtain

\[
\left( f_{01} \right)^2 = \frac{1}{(2\pi)^2 L_1 C} \quad \text{or} \quad C = \frac{1}{(2\pi f_{01})^2 L_1}
\]  \hspace{1cm} (2)
Substituting Equation (2) into the second of Equations (1) yields

\[
f_{02} = \frac{1}{2\pi \sqrt{L_2 C}} = \frac{1}{2\pi \sqrt{\frac{L_2}{L_1}} \left(\frac{1}{(2\pi f_{01})^2 L_1}\right)} = \frac{2\pi f_{01}}{L_1} \sqrt{\frac{L_1}{L_2}}
\]

(3)

In Equation (3) the initial resonant frequency is multiplied by the square root of the ratio of the inductances, so that if we express the initial resonant frequency in kHz, the final resonant frequency will also be expressed in kHz, as requested. Similarly, we do not need to convert the inductances from millihenries to henries, since their units will cancel out. From Equation (3), then, the final resonant frequency of the circuit is

\[
f_{02} = f_{01} \sqrt{\frac{L_1}{L_2}} = (1.3 \text{ kHz}) \sqrt{\frac{7.0 \text{ mH}}{1.5 \text{ mH}}} = 2.8 \text{ kHz}
\]

37. **REASONING** Since the resonant frequency \(f_0\) is known, we may use Equation 23.10, \(f_0 = \frac{1}{2\pi \sqrt{LC}}\), to find the inductance \(L\), provided the capacitance \(C\) can be determined. The capacitance can be found by using the definitions of capacitive and inductive reactances.

**SOLUTION**

a. Solving Equation 23.10 for the inductance, we have

\[
L = \frac{1}{4\pi^2 f_0^2 C}
\]

(1)

where \(f_0\) is the resonant frequency. From Equations 23.2 and 23.4, the capacitive and inductive reactances are

\[
X_C = \frac{1}{2\pi f C} \quad \text{and} \quad X_L = 2\pi f L
\]

where \(f\) is any frequency. Solving the first of these equations for \(f\), substituting the result into the second equation, and solving for \(C\) yields \(C = \frac{L}{X_L X_C}\). Substituting this result into Equation (1) above and solving for \(L\) gives

\[
L = \frac{1}{2\pi f_0} \sqrt{X_L X_C} = \frac{1}{2\pi} \frac{1}{500 \text{ Hz}} \sqrt{20.0 \Omega \cdot 20 \Omega} = 1.3 \times 10^{-3} \text{ H}
\]

b. The capacitance is

\[
C = \frac{L}{X_L X_C} = \frac{1.3 \times 10^{-3} \text{ H}}{20.0 \Omega \cdot 20 \Omega} = 8.7 \times 10^{-6} \text{ F}
\]
38. **REASONING** The inductive reactance \( X_L \) is given by \( X_L = 2\pi f L \) (Equation 23.4), where \( f \) is the nonresonant frequency in hertz (Hz) and \( L \) is the inductance. The capacitive reactance \( X_C \) is given by \( X_C = \frac{1}{2\pi f C} \) (Equation 23.2), where \( C \) is the capacitance. The resonant frequency \( f_0 \) is \( f_0 = \frac{1}{2\pi \sqrt{LC}} \) (23.10). From the values given for the ratio 
\[
\left( \frac{X_L}{X_C} = 5.36 \right)
\]
and the resonant frequency \( f_0 = 225 \text{ Hz} \) we will determine the nonresonant frequency \( f \).

**SOLUTION** Using Equation 23.4 for \( X_L \) and Equation 23.2 for \( X_C \), we can write the ratio of the two reactances as follows:

\[
\frac{X_L}{X_C} = \frac{2\pi f L}{1/(2\pi f C)} = 4\pi^2 f^2 LC
\]

Solving Equation (1) for the nonresonant frequency \( f \) shows that

\[
\frac{X_L}{X_C} = 4\pi^2 f^2 LC \quad \text{or} \quad f = \sqrt{\frac{X_L}{X_C} \left( \frac{1}{4\pi^2 LC} \right)} = \sqrt{\frac{X_L}{X_C} \left( \frac{1}{2\pi \sqrt{LC}} \right)} \quad \text{Resonant frequency}
\]

We note in Equation (2) that the term in parentheses is the resonant frequency \( f_0 \) as given by Equation 23.10, so that we have

\[
f = \left( \frac{X_L}{X_C} \right) (f_0) = (\sqrt{5.36}) (225 \text{ Hz}) = 521 \text{ Hz}
\]

39. **SSM REASONING AND SOLUTION** At the resonant frequency \( f_0 \), we have \( C = 1/(4\pi^2 f_0^2 L) \). We want to determine some series combination of capacitors whose equivalent capacitance \( C'_s \) is such that \( f'_0 = 3f_0 \). Thus,

\[
C'_s = \frac{1}{4\pi^2 f'_0^2 L} = \frac{1}{4\pi^2 (3f_0)^2 L} = \frac{1}{9} \cdot \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{9} C
\]

The equivalent capacitance of a series combination of capacitors is \( 1/C'_s = 1/C_s + 1/C_2 + \ldots \). If we require that all the capacitors have the same capacitance \( C \), the equivalent capacitance is

\[
\frac{1}{C'_s} = \frac{1}{C} + \frac{1}{C} + \ldots = \frac{n}{C}
\]

where \( n \) is the total number of identical capacitors. Using the result above, we find that
\[ \frac{1}{C_s'} = \frac{1}{\frac{1}{n}C} = \frac{n}{C} \quad \text{or} \quad n = 9 \]

Therefore, the number of additional capacitors that must be inserted in series in the circuit so that the frequency triples is \( n' = n - 1 = 8 \).

40. **REASONING**

a. When a capacitor stores charge, it also stores electrical energy. The energy stored by the capacitor can be expressed as \( \text{Energy} = \frac{1}{2} \left( \frac{q^2}{C} \right) \), according to Equation 19.11c.

b. There is no resistance in the circuit, so no energy is lost as it shuttles back and forth between the capacitor and the inductor. The energy removed from the capacitor when it is completely discharged is \( \frac{1}{2} \left( \frac{q^2}{C} \right) \). This energy is gained by the inductor. The energy stored by an inductor is given by \( \text{Energy} = \frac{1}{2} L I^2 \) (Equation 22.10), where \( L \) is the inductance and \( I \) is the current. The maximum energy stored by the inductor is \( \frac{1}{2} L I_{\text{max}}^2 \), where \( I_{\text{max}} \) is the maximum current in the inductor.

**SOLUTION**

a. The electrical energy stored in the fully charged capacitor is

\[ \text{Energy} = \frac{1}{2} \left( \frac{q^2}{C} \right) = \frac{1}{2} \left[ \frac{(2.90 \times 10^{-6} \text{ C})^2}{3.60 \times 10^{-6} \text{ F}} \right] = 1.17 \times 10^{-6} \text{ J} \quad (19.11c) \]

b. Since the energy stored by the capacitor is equal to the maximum energy stored by the inductor, we can write

\[ \frac{1}{2} \left( \frac{q^2}{C} \right) = \frac{1}{2} L I_{\text{max}}^2 \quad \text{or} \quad I_{\text{max}} = \frac{q}{\sqrt{LC}} \]

The maximum current in the inductor is

\[ I_{\text{max}} = \frac{q}{\sqrt{LC}} = \frac{2.90 \times 10^{-6} \text{ C}}{\sqrt{(75.0 \times 10^{-3} \text{ H})(3.60 \times 10^{-6} \text{ F})}} = 5.58 \times 10^{-3} \text{ A} \]
41. **Reasoning** We will find the desired percentage from the ratio $L_B / L_A$. The beat frequency that is heard is $|f_{0B} - f_{0A}|$, and the resonant frequencies are $f_{0B} = 1 / (2\pi \sqrt{L_B/C})$ and $f_{0A} = 1 / (2\pi \sqrt{L_A/C})$, according to Equation 23.10. By expressing the beat frequency in terms of these expressions, we will be able to obtain $L_B / L_A$.

**Solution** Using Equation 23.10 to express each resonant frequency, we find that the beat frequency is

$$|f_{0B} - f_{0A}| = \left| \frac{1}{2\pi \sqrt{L_B/C}} - \frac{1}{2\pi \sqrt{L_A/C}} \right|$$

Factoring out the term $1 / (2\pi \sqrt{L_A/C})$ gives

$$|f_{0B} - f_{0A}| = \frac{1}{2\pi \sqrt{L_A/C}} \left| \frac{1}{2\pi \sqrt{L_B/C}} - 1 \right| = \frac{1}{2\pi \sqrt{L_A/C}} \left| \frac{L_A - 1}{L_B} \right| = \frac{1}{2\pi \sqrt{L_A/C}} \left( 1 - \sqrt{\frac{L_A}{L_B}} \right)$$

Note that $L_B$ is greater than $L_A$, so that $\sqrt{L_A/L_B} - 1$ is a negative quantity. Therefore, we have written $\sqrt{L_A/L_B} - 1$ as $\left( 1 - \sqrt{L_A/L_B} \right)$. Solving for $\sqrt{L_A/L_B}$, we obtain

$$\sqrt{\frac{L_A}{L_B}} = 1 - \frac{|f_{0B} - f_{0A}|}{1 / (2\pi \sqrt{L_A/C})}$$

Remembering that $f_{0A} = 1 / (2\pi \sqrt{L_A/C})$, we see that this result becomes

$$\sqrt{\frac{L_A}{L_B}} = 1 - \frac{|f_{0B} - f_{0A}|}{f_{0A}}$$

or

$$\frac{L_A}{L_B} = \left( 1 - \frac{|f_{0B} - f_{0A}|}{f_{0A}} \right)^2$$

Taking the reciprocal of this expression reveals that

$$\frac{L_B}{L_A} = \frac{1}{\left( 1 - \frac{|f_{0B} - f_{0A}|}{f_{0A}} \right)^2} = \frac{1}{\left( 1 - \frac{7.30 \text{ kHz}}{630.0 \text{ kHz}} \right)^2} = 1.024$$

Thus, the percentage increase of $L_B$ is $(1.024 - 1.000) \times 100\% = 2.4\%$. 


42. **REASONING AND SOLUTION** The current in an RCL-circuit is

\[ I = \frac{V}{\sqrt{R^2 + \alpha^2 - \omega^2}} \]

Rearranging terms

\[(X_L - X_C)^2 = (V/I)^2 - R^2\]

Using Equations 23.2 and 23.4 for \(X_C\) and \(X_L\), respectively, we obtain

\[2\pi f L - \frac{1}{2\pi f C} = \sqrt{\left(\frac{V}{I}\right)^2 - R^2} = \sqrt{\left(\frac{26.0 \text{ V}}{0.141 \text{ A}}\right)^2 - (108 \text{ \Omega})^2} = 149 \text{ \Omega}\]

Multiplying by \(f\) leads to

\[2\pi f^2 L - (149 \text{ \Omega}) f - 1/(2\pi C) = 0\]

or

\[2\pi f^2(5.42 \times 10^{-3} \text{ H}) - (149 \text{ \Omega}) f - 1/[(2\pi(0.200 \times 10^{-6} \text{ F})] = 0\]

We can solve this quadratic equation for the frequencies. We obtain

\[f_1 = 3.11 \times 10^3 \text{ Hz} \quad \text{and} \quad f_2 = 7.50 \times 10^3 \text{ Hz}\]

43. **SSM REASONING** The voltage across the capacitor reaches its maximum instantaneous value when the generator voltage reaches its maximum instantaneous value. The maximum value of the capacitor voltage first occurs one-fourth of the way, or one-quarter of a period, through a complete cycle (see the voltage curve in Figure 23.4).

**SOLUTION** The period of the generator is \(T = 1/f = 1/(5.00 \text{ Hz}) = 0.200 \text{ s}\). Therefore, the least amount of time that passes before the instantaneous voltage across the capacitor reaches its maximum value is \(\frac{T}{4} = \frac{1}{4}(0.200 \text{ s}) = 5.00 \times 10^{-2} \text{ s}\).

44. **REASONING** The average power dissipated is that dissipated in the resistor and is \(\overline{P} = I_{\text{rms}}^2 R\), according to Equation 20.15b. We are given the current \(I_{\text{rms}}\) but need to find the resistance \(R\). Since the inductive reactance \(X_L\) is known, we can find the resistance from the impedance, which is \(Z = \sqrt{R^2 + X_L^2}\), according to Equation 23.7. Since the voltage and the current are known, we can obtain the impedance from Equation 23.6 as \(Z = V_{\text{rms}}/I_{\text{rms}}\).

**SOLUTION** From Equation 23.7, we can determine the resistance as \(R = \sqrt{Z^2 - X_L^2}\). With this expression for the resistance, Equation 20.15b for the power becomes
\[ P = I_{\text{rms}}^2 R = I_{\text{rms}}^2 \sqrt{Z^2 - X_L^2} \]

Using Equation 23.6 to express the impedance, we obtain the following value for the dissipated power:

\[ P = I_{\text{rms}}^2 \sqrt{Z^2 - X_L^2} = I_{\text{rms}}^2 \sqrt{\frac{15 \text{ V}}{175 \text{ A}}} \cdot \frac{1}{175 \text{ A}} \cdot X_L = 20.0 \Omega \cdot g = 123 \text{ W} \]

45. **REASONING AND SOLUTION** We begin by calculating the impedance of the circuit using \( Z = \sqrt{R^2 + \omega L - X_C} \). We have:

\[ X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \cdot 1350 \text{ Hz} \cdot 10^{-6} \text{ F}} = 28.8 \Omega \]

\[ X_L = 2\pi f L = 2\pi (1350 \text{ Hz})(5.30 \times 10^{-3} \text{ H}) = 45.0 \Omega \]

\[ Z = \sqrt{65.0 \Omega + 5.0 \Omega - 28.8 \Omega} \cdot g = 22.8 \Omega \]

The current is therefore,

\[ I = \frac{V}{Z} = \frac{15.0 \text{ V}}{22.8 \Omega} = 0.658 \text{ A} \]

Since the circuit elements are in series, the current through each element is the same. The voltage across each element is

\[ V_R = IR = (0.658 \text{ A})(16.0 \Omega) = 10.5 \text{ V} \]

\[ V_C = IX_C = (0.658 \text{ A})(28.8 \Omega) = 19.0 \text{ V} \]

\[ V_L = IX_L = (0.658 \text{ A})(45.0 \Omega) = 29.6 \text{ V} \]

46. **REASONING AND SOLUTION** At very high frequencies the capacitors behave as if they were replaced with wires that have zero resistance, while the inductors behave as if they were cut out of the circuit. The drawings below show the circuits under this condition. Circuit I behaves as if the two resistors are in parallel, and the equivalent resistance can be obtained from \( R_p^{-1} = R_1^{-1} + R_2^{-1} \) as \( R_p = R/2 \). Circuit II behaves as if the two resistors are in series, and the equivalent resistance is \( R_s = R + R = 2R \). In either case, the current is the voltage divided by the resistance. Therefore, the ratio of the currents in the two circuits is
47. **REASONING** The individual reactances are given by Equations 23.2 and 23.4, respectively,

**Capacitive reactance** \( X_C = \frac{1}{2\pi f C} \)

**Inductive reactance** \( X_L = 2\pi f L \)

When the reactances are equal, we have \( X_C = X_L \), from which we find

\[
\frac{1}{2\pi f C} = 2\pi f L \quad \text{or} \quad 4\pi^2 f^2 LC = 1
\]

The last expression may be solved for the frequency \( f \).

**SOLUTION** Solving for \( f \) with \( L = 52 \times 10^{-3} \) H and \( C = 76 \times 10^{-6} \) F, we obtain

\[
f = \frac{1}{2\pi\sqrt{LC}} = \frac{\frac{1}{2\pi\sqrt{(52 \times 10^{-3}) (76 \times 10^{-6})}}}{2\pi(52 \times 10^{-3}) (76 \times 10^{-6})} = 8.0 \times 10^1 \text{ Hz}
\]

48. **REASONING** Only the resistor, on average, consumes power. Therefore, the average power delivered to the circuit is equal to the average power delivered to the resistor. The average power is given by \( \bar{P} = I_{\text{rms}}^2 R \) (Equation 20.15b), where \( I_{\text{rms}} \) is the rms current in the circuit and \( R \) is the resistance. According to Equation 23.6, the current is given by \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \), where \( V_{\text{rms}} \) is the rms voltage of the generator and \( Z \) is the circuit impedance.

The impedance of the circuit is given by \( Z = \sqrt{R^2 + \omega L - X_C \omega} \) (Equation 23.7). At resonance the inductive reactance \( X_L \) and the capacitive reactance \( X_C \) are equal, so that \( Z = R \).
**SOLUTION** Substituting \( I_{\text{rms}} = V_{\text{rms}} / Z \) into \( \overline{P} = I_{\text{rms}}^2 R \), the average power delivered to the circuit can be written as

\[
\overline{P} = I_{\text{rms}}^2 R = \left( \frac{V_{\text{rms}}}{Z} \right)^2 R
\]

Substituting \( Z = R \) into Equation (1) yields

\[
\overline{P} = \left( \frac{V_{\text{rms}}}{Z} \right)^2 R = \left( \frac{V_{\text{rms}}}{R} \right)^2 R = \left( \frac{V_{\text{rms}}}{R} \right)^2 \left( \frac{R}{92 \, \Omega} \right) = \left( \frac{3.0 \, \text{V}}{92 \, \Omega} \right)^2 = 0.098 \, \text{W}
\]

49. **REASONING** The rms current in an inductor is \( I_{\text{rms}} = V_{\text{rms}} / X_L \), according to Equation 23.3. The inductive reactance is \( X_L = 2\pi f L \), according to Equation 23.4. Applying these expressions to both generators will allow us to obtain the desired current.

**SOLUTION** Using Equations 23.3 and 23.4, we find that the current in an inductor is

\[
I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{V_{\text{rms}}}{2\pi f L}
\]

Applying this result to the two generators gives

\[
I_1 = \frac{V_{\text{rms}}}{2\pi f_1 L} \quad \text{and} \quad I_2 = \frac{V_{\text{rms}}}{2\pi f_2 L}
\]

Generator 1

Generator 2

Dividing the equation for generator 2 by the equation for generator 1, we obtain

\[
\frac{I_2}{I_1} = \frac{V_{\text{rms}}}{2\pi f_2 L} \cdot \frac{2\pi f_1 L}{V_{\text{rms}}} = \frac{f_1}{f_2} \quad \text{or} \quad I_2 = I_1 \frac{f_1}{f_2} = 0.30 \, \text{A} \cdot \frac{1.5 \, \text{kHz}}{0.0 \, \text{kHz}} = 0.075 \, \text{A}
\]

50. **REASONING** To find the frequency at which the current is one-half its value at zero frequency, we first determine the value of the current when \( f = 0 \) Hz. We note that at zero frequency the reactive inductance is zero \( (X_L = 0 \, \Omega) \), since \( X_L = 2\pi f L \) (Equation 23.4). The current at zero frequency is \( I_{\text{rms}}^0 = V_{\text{rms}} / R \) (Equation 20.14), since the inductor does not play a role in determining the current at this frequency. When the frequency is not zero, the current is given by \( I_{\text{rms}} = V_{\text{rms}} / Z \) (Equation 23.6), where \( Z \) is the impedance of the circuit.
These last two relations will allow us to find the frequency at which the current is one-half its value at zero frequency.

**SOLUTION** We are given that \( I_{\text{rms}} = \frac{1}{2} I_{\text{rms}}^0 \), so

\[
\frac{V_{\text{rms}}}{Z} = \frac{1}{2} \left( \frac{V_{\text{rms}}}{R} \right) \quad \text{or} \quad Z = 2R
\]

Since the impedance of the circuit is \( Z = \sqrt{R^2 + X_L^2} \) (Equation 23.7) and \( X_L = 2\pi f L \) (Equation 23.4), the relation \( Z = 2R \) becomes

\[
\sqrt{R^2 + (2\pi f L)^2} = 2R
\]

Solving for the frequency gives

\[
f = \frac{\sqrt{3} R}{2\pi L} = \frac{\sqrt{3} (16 \, \Omega)}{2\pi (4.0 \times 10^{-3} \, \text{H})} = 1.1 \times 10^3 \, \text{Hz}
\]

51. **SSM REASONING** Since we know the values of the resonant frequency of the circuit, the capacitance, and the generator voltage, we can find the value of the inductance from Equation 23.10, the expression for the resonant frequency. The resistance can be found from energy considerations at resonance; the power factor is given by \( \cos \phi \), where the phase angle \( \phi \) is given by Equation 23.8, \( \tan \phi = (X_L - X_C) / R \).

**SOLUTION**

a. Solving Equation 23.10 for the inductance \( L \), we find that

\[
L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 (1.30 \times 10^3 \, \text{Hz})^2 (5.10 \times 10^{-6} \, \text{F})} = 2.94 \times 10^{-3} \, \text{H}
\]

b. At resonance, \( f = f_0 \), and the current is a maximum. This occurs when \( X_L = X_C \), so that \( Z = R \). Thus, the average power \( \bar{P} \) provided by the generator is \( \bar{P} = \frac{V_{\text{rms}}^2}{R} \), and solving for \( R \) we find

\[
R = \frac{V_{\text{rms}}^2}{\bar{P}} = \frac{(11.0 \, \text{V})^2}{25.0 \, \text{W}} = 4.84 \, \Omega
\]

c. When the generator frequency is 2.31 kHz, the individual reactances are
\[ X_C = \frac{1}{2 \pi f C} = \frac{1}{2 \pi (2.31 \times 10^3 \text{ Hz})(5.10 \times 10^{-6} \text{ F})} = 13.5 \Omega \]

\[ X_L = 2 \pi f L = 2 \pi (2.31 \times 10^3 \text{ Hz})(2.94 \times 10^{-3} \text{ H}) = 42.7 \Omega \]

The phase angle \( \phi \) is, from Equation 23.8,

\[
\phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) = \tan^{-1} \left( \frac{42.7 \Omega - 13.5 \Omega}{4.84 \Omega} \right) = 80.6^\circ
\]

The power factor is then given by

\[ \cos \phi = \cos 80.6^\circ = 0.163 \]

52. **REASONING AND SOLUTION** With only the resistor in the circuit, the power dissipated is \( P_1 = V_0^2/R = 1.000 \text{ W} \). Therefore, \( V_0^2 = (1.000 \text{ W}) R \). When the capacitor is added in series with the resistor, the power dissipated is given by \( P_2 = I_2 V_0 \cos \phi = 0.500 \text{ W} \), where \( \cos \phi \) is the power factor, with \( \cos \phi = R/Z_2 \), and \( I_2 = V_0/Z_2 \). The impedance \( Z_2 \) is

\[ Z_2 = \sqrt{R^2 + X_C^2} \]. Substituting yields,

\[ P_2 = V_0^2 R/Z_2^2 = (1.000 \text{ W}) R^2/(R^2 + X_C^2) = 0.500 \text{ W} \]

Solving for \( X_C \) gives, \( X_C = R \). When the inductor is added in series with the resistor, we have \( P_3 = V_0 I_3 \cos \phi = 0.250 \text{ W} \), where \( I_3 = V_0/Z_3 \) and \( \cos \phi = R/Z_3 \). The impedance \( Z_3 \) is

\[ Z_3 = \sqrt{R^2 + X_L^2} \]. Thus,

\[ P_3 = (1.000 \text{ W}) R^2/(R^2 + X_L^2) = 0.250 \text{ W} \]

Solving for \( X_L \), we find that \( X_L = R \sqrt{3} \). Finally, when both the inductor and capacitor are added in series with the resistor we have

\[ P_4 = \frac{V_0^2 R}{R^2 + (X_L - X_C)^2} = \frac{(1.000 \text{ W}) R^2}{R^2 + (R \sqrt{3} - R)^2} = 0.651 \text{ W} \]
CHAPTER 24 | ELECTROMAGNETIC WAVES

ANSWERS TO FOCUS ON CONCEPTS QUESTIONS

1. (b) The loop can only detect the wave if the wave’s magnetic field has a component perpendicular to the plane of the loop, that is, along the y axis. Only then will there be a changing magnetic flux through the loop. The changing flux is needed, so that an induced emf will arise in the loop according to Faraday’s law of electromagnetic induction. The electric and magnetic fields of an electromagnetic wave are mutually perpendicular and are both perpendicular to the direction in which the wave travels. Thus, when the wave travels along the z axis with its electric field along the x axis, the magnetic field will be along the y axis as needed.

2. (c) The wavelength $\lambda$, frequency $f$, and speed $c$ of an electromagnetic wave are related according to $c = \lambda f$, where $c$ is the same for any electromagnetic wave traveling in a vacuum and is independent of $\lambda$ and $f$. Since $c$ is constant, $\lambda$ and $f$ are inversely proportional. When $f$ is reduced by a factor of three, $\lambda$ increases by a factor of three.

3. (b) The magnitudes of the electric and magnetic fields of the wave are proportional to each other, according to $E = cB$ (Equation 24.3). As Section 24.4 discusses, the wave carries equal amounts of electric and magnetic energy.

4. (a) The magnitudes of the electric and magnetic fields of the wave are proportional, according to $E = cB$ (Equation 24.3). Thus, when $E$ doubles, so does $B$. The total energy density and the intensity are each proportional to the square of the electric field magnitude, according to $u = \varepsilon_0 E^2$ (Equation 24.2b) and $S = c\varepsilon_0 E^2$ (Equation 24.5b). Therefore, when $E$ doubles, $u$ and $S$ both increase by a factor of $2^2 = 4$.

5. 698 J/(s·m²)

6. (d) The observed frequency is $f_o = f_s \left(1 \pm \frac{v_{rel}}{c}\right)$ according to Equation 24.6. The frequency $f_s$ emitted by the source is the same in each case, so that only the relative speed $v_{rel}$ and the direction of the relative motion determine the observed frequency. In each case either the source or the observer is moving, so the relative speed is just the magnitude of the velocity vector shown in the drawing. Since the velocity vector has the same magnitude in each case, the relative speed is the same in each case. Thus, it is only the direction of the relative motion that needs to be considered here. In A and C the source and observer are moving apart at the same relative speed, the minus sign applies in Equation 24.6, and the observed frequencies are the same. In B and D they are coming together at the same relative speed, the plus sign applies in Equation 24.6, and the observed frequencies are the same, but greater than that in A and C.
7. $3.0 \times 10^6$ m/s

8. (c) When the incident light is completely unpolarized, half of its intensity is absorbed by the polarizer on the left, and half passes through. The half that passes through is completely polarized along the vertical direction, which is the same as the transmission axis of the second polarizer. Thus, the second polarizer absorbs none of the light, and the intensity of the exiting light is half that of the incident light. When the incident light is completely polarized along the vertical direction to begin with, it passes through both sheets of material with none of its intensity being absorbed. In this case the exiting light has the same intensity as the incident light. Thus, the exiting light has a greater intensity when the incident light is polarized.

9. (d) When the unpolarized light strikes the first polarizer, the light that passes through it is polarized in the vertical direction. When this polarized light strikes the second polarizer, all of it is absorbed, since the two polarizers are crossed. When the polarized light strikes the first polarizer, all of it passes through, since the direction of polarization and the transmission axis are both vertical. When this polarized light strikes the second polarizer, all of it is absorbed, since the two polarizers are crossed. Thus, no light exits the polarizer on the right in either case.

10. (e) When the light is incident from the left, Malus' law (Equation 24.7) indicates that the transmitted light has an average intensity that is reduced relative to the incident intensity by a factor of $\cos^2 45^\circ = \frac{1}{2}$. When the light is incident from the right, Malus' law also applies. Now, however, polarizer 2 has its transmission axis at an angle of $45^\circ$ with respect to the polarization direction of the incident light. Malus' law indicates that the intensity is reduced by a factor of $\frac{1}{2}$. But the light leaving polarizer 2 is polarized at an angle of $45^\circ$ with respect to the transmission axis of polarizer 1. Malus' law again applies and indicates that the intensity is reduced by a second factor of $\frac{1}{2}$. The transmitted light, therefore, has an intensity that is reduced by a factor of $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ relative to the initial intensity.
CHAPTER 24 | ELECTROMAGNETIC WAVES

PROBLEMS

1. **REASONING** The distance $d$ between earth and the probe is determined by $d = ct$ (Equation 2.1), where $c$ is the speed of light in a vacuum and $t$ is the time for the radio signal to reach earth.

   **SOLUTION** The elapsed time $t$ is given in hours, so it must be converted to seconds:
   
   $$t = (2.53 \text{ h}) \left( \frac{3600 \text{ s}}{1 \text{ h}} \right) = 9110 \text{ s} \quad (1)$$
   
   The distance, then, between earth and the probe is
   
   $$d = ct = (3.00 \times 10^8 \text{ m/s})(9110 \text{ s}) = 2.73 \times 10^{12} \text{ m}$$

2. **REASONING** Radio waves are electromagnetic waves that travel at the speed of light. In the vacuum of space light travels at a speed of $c = 3.00 \times 10^8 \text{ m/s}$. The constant speed $c$ is given by $c = \frac{d}{t}$ (Equation 2.1), where $d$ is the distance traveled in a time $t$. We can use this equation to determine $t$, since values are known for $c$ and $d$.

   **SOLUTION**
   a. Solving Equation 2.1 for the communication time $t$ between the moon and the earth reveals that
   
   $$t = \frac{d}{c} = \frac{3.85 \times 10^8 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 1.28 \text{ s}$$
   
   b. A calculation similar to that in part a shows that the minimum communication time between Mars and the earth is
   
   $$t = \frac{d}{c} = \frac{5.6 \times 10^{10} \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 190 \text{ s}$$

3. **SSM REASONING** This is a standard exercise in the conversion of units. However, we first need to determine the number of meters that light travels in one year, which is what we call a light-year. In the vacuum of space light travels at a speed of $c = 3.00 \times 10^8 \text{ m/s}$. The constant speed $c$ is given by $c = \frac{d}{t}$ (Equation 2.1), where $d$ is the distance traveled in a time $t$. We can use this equation to determine $d$, since values are known for $c$ and $t$. Then,
armed with this value for one light-year, we will follow the usual procedure for converting units, as discussed in Section 1.3.

**SOLUTION** Solving Equation 2.1 for the distance \( d \) that light travels in one year, we find

\[
d = ct = \left( 3.00 \times 10^8 \text{ m/s} \right) \left( 1.00 \text{ year} \right) \left( 365.25 \text{ days} \right) \left( 24 \text{ hours} \right) \left( 3600 \text{ s} \right) = 9.47 \times 10^{15} \text{ m}
\]

Conversion of 1.00 year into seconds

Thus, 1 light-year = \( 9.47 \times 10^{15} \) m. To find the distance to Alpha Centauri in meters, we follow the usual procedure and multiply the distance of 4.3 light-years by

\[
1 = \frac{9.47 \times 10^{15} \text{ m}}{1 \text{ light-year}}
\]

In so doing, we obtain

\[
\left( 4.3 \text{ light-years} \right) \left( \frac{9.47 \times 10^{15} \text{ m}}{1 \text{ light-year}} \right) = 4.1 \times 10^{16} \text{ m}
\]

4. **REASONING** In order to pick up radio waves, the circuit must have a resonant frequency \( f_0 \) that matches the frequency of the radio waves. The resonant frequency depends upon the capacitance \( C \) and inductance \( L \) of the circuit via \( f_0 = \frac{1}{2\pi \sqrt{LC}} \) (Equation 23.10). In order to pick up the entire range of FM waves, the circuit must be able to attain the lowest \( (f_{\text{low}} = 88 \text{ MHz}) \) and highest \( (f_{\text{high}} = 108 \text{ MHz}) \) necessary resonant frequency. We will use Equation 23.10 to determine the corresponding minimum and maximum capacitance values.

**SOLUTION** Squaring both sides of \( f_0 = \frac{1}{2\pi \sqrt{LC}} \) (Equation 23.10) and solving for \( C \), we obtain

\[
(f_0)^2 = \frac{1}{(2\pi)^2 LC} \quad \text{or} \quad C = \frac{1}{(2\pi f_0)^2 L}
\]

(1)

As we see from Equation (1), the greater the frequency, the smaller the value of the capacitance. So the highest frequency \( f_{\text{high}} = 108 \text{ MHz} \) corresponds to the minimum value of the capacitance \( C_{\text{min}} \). From Equation (1), we obtain

\[
C_{\text{min}} = \frac{1}{\left( 2\pi f_{\text{high}} \right)^2 L} = \frac{1}{\left( 2\pi \right)^2 (108 \times 10^6 \text{ Hz})^2 \left( 6.00 \times 10^{-7} \text{ H} \right)} = 3.62 \times 10^{-12} \text{ F}
\]

On the other end of the FM frequency range, matching the lowest frequency of \( f_{\text{low}} = 88.0 \text{ MHz} \) requires a maximum capacitance value \( C_{\text{max}} \) of
\[ C_{\text{max}} = \frac{1}{(2\pi f_{\text{low}})^2 L} = \frac{1}{(2\pi)^2 \left(88.0 \times 10^6 \text{ Hz}\right)^2 \left(6.00 \times 10^{-7} \text{ H}\right)} = 5.45 \times 10^{-12} \text{ F} \]

Therefore, the capacitance values should range from \(3.62 \times 10^{-12} \text{ F}\) to \(5.45 \times 10^{-12} \text{ F}\).

5. **REASONING** According to Equation 16.3, the displacement \(y\) of a wave that travels in the +x direction and has amplitude \(A\), frequency \(f\), and wavelength \(\lambda\) is given by

\[ y = A \sin \left(2\pi f t - \frac{2\pi x}{\lambda}\right) \]

This equation, with \(y = E\), applies to the traveling electromagnetic wave in the problem, which is represented mathematically as

\[ E = E_0 \sin \left[ \left(1.5 \times 10^{10} \text{ s}^{-1}\right)t - \left(5.0 \times 10^1 \text{ m}^{-1}\right)x\right] \]

As \(E_0\) is the maximum field strength, it represents the amplitude \(A\) of the wave. We can find the frequency and wavelength of this electromagnetic wave by comparing the mathematical form of the electric field with Equation 16.3.

**SOLUTION**

a. By inspection, we see that \(2\pi f = 1.5 \times 10^{10} \text{ s}^{-1}\). Therefore, the frequency of the wave is

\[ f = \frac{1.5 \times 10^{10} \text{ s}^{-1}}{2\pi} = \frac{2.4 \times 10^9}{2\pi} \text{ Hz} \]

b. As shown in Figure 17.15, the separation between adjacent nodes in any standing wave is one-half of a wavelength. By inspection of the mathematical form of the electric field and comparison with Equation 16.3, we infer that \(2\pi / \lambda = 5.0 \times 10^1 \text{ m}^{-1}\). Therefore,

\[ \lambda = \frac{2\pi}{5.0 \times 10^1 \text{ m}^{-1}} = 0.126 \text{ m} \]

Therefore, the nodes in the standing waves formed by this electromagnetic wave are separated by \(\lambda/2 = 0.063 \text{ m}\).

6. **REASONING AND SOLUTION** The average flux change through the coil in one fourth of the wave period is, according to Faraday's law, \(\Delta \Phi = NBA = NB_0 A\). The magnitude of the average emf is then \(\text{emf} = \Delta \Phi / \Delta t = NB_0 A / \Delta t\). Now \(\Delta t = T/4 = 1/(4f)\), so

\[ \text{Emf} = 4NfB_0 A = 4(450)(1.2 \times 10^6 \text{ Hz})(2.0 \times 10^{-13} \text{ T})(\pi)(0.25 \text{ m})^2 = 8.5 \times 10^{-5} \text{ V} \]
7. **REASONING AND SOLUTION** Using Equation 16.1, we obtain

\[ \lambda = \frac{c}{f} = \frac{2.9979 \times 10^8 \text{ m/s}}{26.965 \times 10^6 \text{ Hz}} = 11.118 \text{ m} \]

8. **REASONING** According to Equation 16.1, the wavelength \( \lambda \) (in vacuum) is the speed of light \( c \) in a vacuum divided by the frequency \( f \) of the X-rays: \( \lambda = \frac{c}{f} \).

**SOLUTION** Using Equation 16.1, we find that

\[ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.05 \times 10^{18} \text{ Hz}} = 4.96 \times 10^{-11} \text{ m} \]

9. **SSM REASONING** The frequency \( f \) of the UHF wave is related to its wavelength by \( c = f \lambda \) (Equation 16.1), where \( c \) is the speed of light in a vacuum and \( \lambda \) is the wavelength. The electric and magnetic fields are both zero at the same positions, which are separated by a distance \( d \) equal to half a wavelength (see Figure 24.3). Therefore, we can express the wavelength in terms of the distance between adjacent positions of zero field as

\[ \lambda = 2d \] (1)

**SOLUTION** Solving \( c = f \lambda \) (Equation 16.1) for \( f \) yields

\[ f = \frac{c}{\lambda} \] (2)

Substituting Equation (1) into Equation (2), we obtain

\[ f = \frac{c}{2d} = \frac{3.00 \times 10^8 \text{ m/s}}{2(0.34 \text{ m})} = 4.4 \times 10^8 \text{ Hz} \]

10. **REASONING** According to Equation 16.1, the wavelength \( \lambda \) (in vacuum) is the speed of light \( c \) in a vacuum divided by the frequency \( f \) of the radio waves: \( \lambda = \frac{c}{f} \).

**SOLUTION** Using Equation 16.1, we find that the longest FM radio wavelength is

\[ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{88.0 \times 10^6 \text{ Hz}} = 3.41 \text{ m} \]
The shortest FM radio wavelength is
\[ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{108.0 \times 10^6 \text{ Hz}} = 2.78 \text{ m} \]

11. **REASONING AND SOLUTION** According to Equation 16.1, the wavelength of these waves is \( \lambda = c/f \). Therefore,
\[ \frac{\lambda_{\text{MRI}}}{\lambda_{\text{PET}}} = \frac{c/f_{\text{MRI}}}{c/f_{\text{PET}}} = \frac{f_{\text{PET}}}{f_{\text{MRI}}} = \frac{1.23 \times 10^{20} \text{ Hz}}{6.38 \times 10^7 \text{ Hz}} = 1.93 \times 10^{12} \]

12. **REASONING** The length of each pulse is equal to the product of its speed and the time, or \( x = ct_0 \). According to Equation 16.1, the wavelength \( \lambda \) in a vacuum is related to the frequency \( f \) by \( \lambda = c/f \), where \( c \) is the speed of light in a vacuum. When the light travels in water, the speed is no longer \( c \), but its value \( v \) is given. The wavelength in water, then, is \( \lambda = v/f \). The frequency in a vacuum and in water is the same.

**SOLUTION**

a. The number of wavelengths in one pulse is equal to the length of the pulse divided by the wavelength. The length of each pulse is \( x = ct_0 \) and the wavelength is \( \lambda = c/f \), so
\[ \text{Number of wavelengths} = \frac{x}{\lambda} = \frac{ct_0}{c/f} = \frac{ct_0}{f} = 5.2 \times 10^{14} \text{ Hz} \left( 2.7 \times 10^{-11} \text{ s} \right) = 1.4 \times 10^4 \]

b. When the light is traveling in water, its speed is \( v \), which is less than the speed of light in a vacuum. The length of each pulse is now \( x = vt_0 \) and the wavelength is \( \lambda = v/f \), so
\[ \text{Number of wavelengths} = \frac{x}{\lambda} = \frac{vt_0}{v/f} = \frac{vt_0}{f} = 5.2 \times 10^{14} \text{ Hz} \left( 2.7 \times 10^{-11} \text{ s} \right) = 1.4 \times 10^4 \]

13. **REASONING** To determine the difference in frequencies, we will calculate each frequency and subtract one from the other. Each frequency \( f \) is related to the wavelength \( \lambda \) and the speed of light \( c \) according to \( f = c/\lambda \) (Equation 16.1).

**SOLUTION** Using Equation 16.1 to calculate each frequency, we find that
\[ f_2 - f_1 = \frac{c}{\lambda_2} - \frac{c}{\lambda_1} = \left( 2.9979 \times 10^8 \text{ m/s} \right) \left( \frac{1}{0.34339 \text{ m}} - \frac{1}{0.36205 \text{ m}} \right) = 4.500 \times 10^7 \text{ Hz} \]
14. **REASONING** According to $\omega = \sqrt{\frac{k}{m}}$ (Equation 10.11), the angular oscillation frequency $\omega$ depends upon the mass $m$ of the oscillator and the spring constant $k$. The angular frequency $\omega$ (in rad/s) of the motion of the oscillating mass is related to the frequency $f$ (in Hz) of the resulting ELF radio waves by $\omega = 2\pi f$ (Equation 10.6). We will use $c = f \lambda$ (Equation 16.1) to determine the frequency of the ELF radio waves, where $c$ is the speed of light in a vacuum and $\lambda$ is the wavelength.

**SOLUTION** Squaring both sides of $\omega = \sqrt{\frac{k}{m}}$ (Equation 10.11) and solving for $k$, we obtain

$$\frac{k}{m} = \omega^2 \quad \text{or} \quad k = m\omega^2 \quad (1)$$

Substituting $\omega = 2\pi f$ (Equation 10.6) into Equation (1) yields

$$k = m(2\pi f)^2 = 4\pi^2 mf^2 \quad (2)$$

Solving $c = f \lambda$ (Equation 16.1) for $f$ gives $f = \frac{c}{\lambda}$. Substituting this result into Equation (2), we find that

$$k = 4\pi^2 m\left(\frac{c}{\lambda}\right)^2 = 4\pi^2 (0.115 \text{ kg})\left(\frac{3.00 \times 10^8 \text{ m/s}}{4.80 \times 10^7 \text{ m}}\right)^2 = 177 \text{ N/m}$$

15. **REASONING** The distance between Polaris and Earth is equal to the speed of the light multiplied by the time it takes for the light to make the journey. The time is given. Since light is an electromagnetic wave, and all electromagnetic waves travel through a vacuum at the speed of light $c$, the speed of the light is also known.

**SOLUTION** The distance $s$ between Polaris and Earth is $s = ct$, where $t$ is the time for the light to travel this distance. Using the fact that $1 \text{ yr} = 3.156 \times 10^7 \text{ s}$ (see the table of conversion factors at the front of the book), we find that

$$s = ct = (3.00 \times 10^8 \text{ m/s}) (680 \text{ yr}) \left(\frac{3.156 \times 10^7 \text{ s}}{1 \text{ yr}}\right) = 6.4 \times 10^{18} \text{ m}$$

16. **REASONING** The mirror must rotate with a sufficient angular speed $\omega$ so that, after reflecting light from the source toward the fixed mirror, one of its faces is in the correct position to intercept light returning from the fixed mirror and reflect it toward the observer. Since the rotating mirror in Michelson’s setup has eight sides, the minimum angular displacement $\Delta \theta$ meeting this condition is one eighth of a revolution: $\Delta \theta = 0.125 \text{ rev}$. The mirror must rotate at least this far in the time $\Delta t$ it takes the light to travel to the fixed mirror.
and back. The minimum, constant, angular speed of the mirror, then, can be found from
\[ \omega = \frac{\Delta \theta}{\Delta t} \] (Equation 8.2). The time \( \Delta t \) it takes the light to travel from the rotating mirror to
the fixed mirror and back is given by \( c = \frac{2d}{\Delta t} \) (Equation 2.1), where \( c = 3.00 \times 10^8 \text{ m/s} \) is the
speed of light in a vacuum, and \( d = 35 \text{ km} \) is the distance between the rotating mirror and
the fixed mirror. Together, Equations 8.2 and 2.1 will allow us to determine the minimum angular speed \( \omega \) of the rotating mirror. The angles between the rays of light shown in Figure
24.12 are exaggerated. In reality, the diameter of the rotating mirror is so much smaller than the
distance \( d \) to the fixed mirror that these two rays may be considered to be parallel.

**SOLUTION** Solving \( c = \frac{2d}{\Delta t} \) (Equation 2.1) for \( \Delta t \) yields
\[ \Delta t = \frac{2d}{c} \] (1)
Substituting Equation (1) into \( \omega = \frac{\Delta \theta}{\Delta t} \) (Equation 8.2), we obtain
\[ \omega = \frac{\Delta \theta}{\Delta t} = \frac{\frac{\Delta \theta}{2d}}{2d} = \frac{\frac{3.00 \times 10^8 \text{ m/s}}{(0.125 \text{ rev})}}{2(35 \times 10^3 \text{ m})} = 540 \text{ rev/s} \]

17. **REASONING** We proceed by first finding the time \( t \) for sound waves to travel
between the astronauts. Since this is the same time it takes for the electromagnetic waves to
travel to earth, the distance between earth and the spaceship is \( d_{\text{earth-ship}} = ct \).

**SOLUTION** The time it takes for sound waves to travel at 343 m/s through the air between
the astronauts is
\[ t = \frac{d_{\text{astronaut}}}{v_{\text{sound}}} = \frac{1.5 \text{ m}}{343 \text{ m/s}} = 4.4 \times 10^{-3} \text{ s} \]
Therefore, the distance between the earth and the spaceship is
\[ d_{\text{earth-ship}} = ct = (3.0 \times 10^8 \text{ m/s})(4.4 \times 10^{-3} \text{ s}) = 1.3 \times 10^6 \text{ m} \]

18. **REASONING** Let \( R \) denote the average rate at which the laptop downloads information,
measured in bits per second (bps). This average rate is equal to the number \( N \) of bits
downloaded in a time \( t \) divided by the time: \( R = N/t \). Therefore, the number \( N \) of bits that the
laptop downloads is given by
\[ N = Rt \] (1)
We note that 1 Mbps (megabit per second) is equal to \( 10^6 \) bps. The time \( t \) is the time it takes
the wireless signal to travel the distance \( d = 8.1 \text{ m} \) between the router and the laptop. This
time is determined by Equation 2.1 as
\[ t = \frac{d}{c} \]  \hspace{1cm} (2)

where \( c = 3.00 \times 10^8 \text{ m/s} \) is the speed of light in a vacuum.

**SOLUTION** Substituting Equation (2) into Equation (1), we obtain

\[ N = R \cdot t = R \left( \frac{d}{c} \right) \]  \hspace{1cm} (3)

To convert the download rate \( R \) into bits per second, we use the equivalence \( 1 \text{ Mbps} = 10^6 \text{ bps}, \) and rewrite “bps” as “bits/s”

\[ R = \left( 260 \text{ Mbps} \right) \frac{10^6 \text{ bps}}{1 \text{ Mbps}} = 260 \times 10^6 \text{ bps} = 260 \times 10^6 \text{ bit/s} \]

Therefore, from Equation (3), the average number \( N \) of bits downloaded is

\[ N = R \left( \frac{d}{c} \right) = \left( 260 \times 10^6 \text{ bits/s} \right) \left( \frac{8.1 \text{ m}}{3.00 \times 10^8 \text{ m/s}} \right) = 7.0 \text{ bits} \]

19. **REASONING** Since the speed at which an electromagnetic wave travels is known, the round-trip travel time \( t \) of a single pulse from the lidar gun can be used to determine the distance \( d \) of the speeding vehicle from the gun as \( d = ct/2 \). Here, we have divided by a factor of two in order to account for the fact that the time given is that for the electromagnetic wave to travel out to the vehicle and return. Thus, in effect, the two pulses are used to measure the distances of the vehicle from the gun at two different instants. The difference in the distances is the distance that the speeder travels in the interval between the pulses. We can determine the vehicle’s speed by dividing the travel distance by the time interval of 0.450 s.

**SOLUTION** Applying the expression \( d = ct/2 \) for each pulse, we obtain the distance \( D \) traveled by the speeding vehicle between the two pulses as

\[ D = d_2 - d_1 = c \left( \frac{1}{2} t_2 \right) - c \left( \frac{1}{2} t_1 \right) \]

Dividing this distance by the interval \( t_{\text{pulses}} \) between the pulses gives the speed \( v \) of the vehicle as

\[ v = \frac{D}{t_{\text{pulses}}} = \frac{c \left( \frac{1}{2} t_2 \right) - c \left( \frac{1}{2} t_1 \right)}{t_{\text{pulses}}} = \frac{1}{2} c \left( \frac{t_2 - t_1}{t_{\text{pulses}}} \right) = \frac{1}{2} \left( 3.00 \times 10^8 \text{ m/s} \right) \left( \frac{1.27 \times 10^{-7} \text{ s}}{0.450 \text{ s}} \right) = 42.3 \text{ m/s} \]

20. **REASONING** The press conference is being broadcast via electromagnetic waves that travel at the speed of light \( c = 3.00 \times 10^8 \text{ m/s} \). According to the stated assumptions and to Equation 2.1, the maximum distance \( d \) that these waves travel between the politician and
viewer's television set is \( d = ct_{\text{em}} \), where \( t_{\text{em}} \) is the travel time of the waves in reaching the television set. We are given no value for \( t_{\text{em}} \). However, we know that the total time required for the sound emitted by the politician to reach the viewer's ears consists of \( t_{\text{em}} \) plus the travel time \( t_{\text{view}} \) of the sound waves emitted by the television set to reach the viewer's ears. We also know that the television viewer and the reporter hear the politician's words at exactly the same instant. Therefore, \( t_{\text{em}} + t_{\text{view}} = t_{\text{rep}} \), where \( t_{\text{rep}} \) is the travel time of the sound waves emitted by the politician to reach the reporter's ears. We can rearrange this equation in order to obtain the necessary value of \( t_{\text{em}} \), namely \( t_{\text{em}} = t_{\text{rep}} - t_{\text{view}} \). To calculate the travel times \( t_{\text{rep}} \) and \( t_{\text{view}} \) of the sound waves through the air, we will use Equation 2.1 in the forms \( d_{\text{rep}} = v_{\text{sound}} t_{\text{rep}} \) and \( d_{\text{view}} = v_{\text{sound}} t_{\text{view}} \), where \( v_{\text{sound}} = 343 \, \text{m/s} \), \( d_{\text{rep}} = 4.1 \, \text{m} \), and \( d_{\text{view}} = 2.3 \, \text{m} \).

**SOLUTION** As discussed in the **REASONING**, the maximum distance \( d \) between the television set and the politician is

\[
d = ct_{\text{em}}
\]  
(1)

As also discussed in the **REASONING**, the travel time \( t_{\text{em}} \) of the electromagnetic waves being used to broadcast the press conference is

\[
t_{\text{em}} = t_{\text{rep}} - t_{\text{view}}
\]  
(2)

Using Equation 2.1 in the forms \( d_{\text{rep}} = v_{\text{sound}} t_{\text{rep}} \) and \( d_{\text{view}} = v_{\text{sound}} t_{\text{view}} \), we can calculate the travel times \( t_{\text{rep}} \) and \( t_{\text{view}} \) of the sound waves through the air as follows:

\[
t_{\text{rep}} = \frac{d_{\text{rep}}}{v_{\text{sound}}} \quad \text{and} \quad t_{\text{view}} = \frac{d_{\text{view}}}{v_{\text{sound}}}
\]  
(3)

Using Equation (2) and Equations (3), we find that Equation (1) becomes

\[
d = ct_{\text{em}} = c \left( t_{\text{rep}} - t_{\text{view}} \right) = c \left( \frac{d_{\text{rep}}}{v_{\text{sound}}} - \frac{d_{\text{view}}}{v_{\text{sound}}} \right)
\]

\[
= \left( 3.00 \times 10^8 \, \text{m/s} \right) \left( \frac{4.1 \, \text{m}}{343 \, \text{m/s}} - \frac{2.3 \, \text{m}}{343 \, \text{m/s}} \right) = 1.6 \times 10^6 \, \text{m}
\]

21. **REASONING** Because the flash from the gunshot travels at the speed \( c = 3.00 \times 10^8 \, \text{m/s} \) of light in a vacuum, it can make \( N \) round trips between the two mirrors in the time \( \Delta t_f \) that it takes the sound of the gunshot to make one round trip, returning as an echo. Therefore, in terms of the time \( \Delta t_f \) for one round-trip of the light flash, the number \( N \) of round trips of the flash is given by
\[ N = \frac{\Delta t_s}{\Delta t_f} \]  

The time \( \Delta t \) it takes either the sound or the flash to travel the round-trip distance \( d \) between the gun and the cliff at a constant speed \( v \) is given by \( \Delta t = \frac{d}{v} \) (Equation 2.1). The sound of the gunshot travels at a speed \( v = 343 \text{ m/s} \), so Equation 2.1 yields both

\[ \Delta t_s = \frac{d}{v} \quad \text{Sound of echo} \quad \text{and} \quad \Delta t_f = \frac{d}{c} \quad \text{Light flash} \]  

**SOLUTION** Substituting Equations (2) into Equation (1) yields

\[ N = \frac{\Delta t_s}{\Delta t_f} = \frac{\left( \frac{d}{v} \right)}{\frac{d}{c}} = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{343 \text{ m/s}} = 8.75 \times 10^5 \]

---

22. **REASONING AND SOLUTION** According to Equation 16.8, we have

\[ S = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{1.2 \times 10^{-3} \text{ W}}{\pi (1.0 \times 10^{-3} \text{ m})^2} = 3.8 \times 10^2 \text{ W/m}^2 \]

---

23. **REASONING AND SOLUTION** Using Equations 24.5b and 24.3, we find the following results:

a. \[ E_{\text{rms}} = \frac{S}{c \epsilon_0} = \frac{1.23 \times 10^9 \text{ W/m}^2}{\left(3.00 \times 10^8 \text{ m/s}\right) \left[8.85 \times 10^{-12} \text{ C}^2/\left(\text{N} \cdot \text{m}^2\right)\right]} = 6.81 \times 10^5 \text{ N/C} \]

b. \[ B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = 2.27 \times 10^{-3} \text{ T} \]

---

24. **REASONING** The magnitude \( E \) of the electric field in an electromagnetic wave is related to the magnitude \( B \) of the magnetic field according to \( E = cB \) (Equation 24.3), where \( c \) is the speed of light.

**SOLUTION** Using Equation 24.3, we find that

\[ E = cB = \left(3.00 \times 10^8 \text{ m/s}\right) \left(3.3 \times 10^{-6} \text{ T}\right) = 990 \text{ N/C} \]
25. **SSM REASONING** The rms value $E_{\text{rms}}$ of the electric field is related to the average energy density $\bar{u}$ of the microwave radiation according to $\bar{u} = \varepsilon_0 E_{\text{rms}}^2$ (Equation 24.2b).

**SOLUTION** Solving for $E_{\text{rms}}$ gives

$$E_{\text{rms}} = \sqrt{\frac{\bar{u}}{\varepsilon_0}} = \sqrt{\frac{4 \times 10^{-14} \text{ J/m}^3}{8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)}} = 0.07 \text{ N/C}$$

26. **REASONING** Since the energy density $u$ is the energy per unit volume of space, the electromagnetic energy contained in a volume $V$ is the product of $u$ and $V$. We are not given a value for $u$. However, the energy density is related to the intensity $S$ of the electromagnetic wave by $u = \frac{S}{c}$ (Equation 24.4), where $c = 3.00 \times 10^8 \text{ m/s}$ is the speed of light in a vacuum.

**SOLUTION** Expressing the energy as the product of the energy density and the volume and using Equation 24.4 to relate the energy density to the intensity, we find that

$$\text{Energy} = uV = \left( \frac{S}{c} \right) V = \left( \frac{1.0 \times 10^3 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s}} \right) (5.5 \text{ m}^3) = 1.8 \times 10^{-5} \text{ J}$$

27. **SSM REASONING** The average intensity of a wave is the average power per unit area that passes perpendicularly through a surface. Thus, the average power $\bar{P}$ of the wave is the product of the average intensity $\bar{S}$ and the area $A$. The average intensity is related to the rms-value $E_{\text{rms}}$ of the electric field by $\bar{S} = c \varepsilon_0 E_{\text{rms}}^2$.

**SOLUTION** The average power is $\bar{P} = \bar{S} A$. Since $\bar{S} = c \varepsilon_0 E_{\text{rms}}^2$ (Equation 24.5b), we have

$$\bar{P} = \bar{S} A = (c \varepsilon_0 E_{\text{rms}}^2) A$$

$$= (3.00 \times 10^8 \text{ m/s}) \left[ 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2) \right] (2.0 \times 10^9 \text{ N/C})^2 (1.6 \times 10^{-5} \text{ m}^2)$$

$$= 1.7 \times 10^{11} \text{ W}$$

28. **REASONING** The average intensity $\bar{S}$ is related to the distance $r$ from the bulb by $\bar{S} = \frac{\bar{P}}{4 \pi r^2}$ (Equation 16.9), where $\bar{P}$ is the average power radiated by the bulb.

The intensity of an electromagnetic wave is related to the magnitude $E$ of its electric field by $S = c \varepsilon_0 E^2$ (Equation 24.5b). According to the discussion in Section 24.4, if the intensity is
an average intensity, then the value for the electric field must be an rms value, not a peak value. The peak value $E_0$ and the rms value $E_{\text{rms}}$ are related by $E_{\text{rms}} = E_0 / \sqrt{2}$.

**SOLUTION**

a. The average intensity of the wave is

$$\bar{S} = \frac{\bar{P}}{4\pi r^2} = \frac{150.0 \text{ W}}{4\pi (5.00 \text{ m})^2} = 0.477 \text{ W/m}^2$$

(Eq. 16.9)

b. The average intensity $\bar{S}$ is related to the rms value $E_{\text{rms}}$ of the electric field by $\bar{S} = c\varepsilon_0 E_{\text{rms}}^2$ (Equation 24.5b). Solving for the electric field gives

$$E_{\text{rms}} = \sqrt{\frac{\bar{S}}{c\varepsilon_0}} = \sqrt{\frac{0.477 \text{ W/m}^2}{(3.00 \times 10^8 \text{ m/s})\left[8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)\right]}} = 13.4 \text{ N/C}$$

c. The rms value $E_{\text{rms}}$ of the electric field is related to the peak value $E_0$ by $E_{\text{rms}} = E_0 / \sqrt{2}$. The peak electric field is, therefore,

$$E_0 = \sqrt{2}E_{\text{rms}} = \sqrt{2}(13.4 \text{ N/C}) = 19.0 \text{ N/C}$$

29. **REASONING AND SOLUTION** Since the sun emits radiation uniformly in all directions, at a distance $r$ from the sun's center, the energy spreads out over a sphere of surface area $4\pi r^2$. Therefore, according to $S = P/(4\pi r^2)$ (Equation 16.9), the total power radiated by the sun is

$$P = S(4\pi r^2) = (1390 \text{ W/m}^2)(4\pi)(1.50 \times 10^{11} \text{ m})^2 = 3.93 \times 10^{26} \text{ W}$$

30. **REASONING** When a stationary charge is placed in an electric field, it experiences an electric force. The magnitude $F$ of the electric force is given by Equation 18.2 as $F = |q|E$, where $|q|$ is the magnitude of the charge and $E$ is the magnitude of the electric field.

When a stationary charge is placed in a magnetic field, it does not experience a magnetic force, because the charge is not moving. According to Equation 21.1, the magnitude of the magnetic force is related to the magnitude $B$ of the magnetic field by $F = |q|vB \sin \theta$, where $v$ is the speed of the charge and $\theta$ is the angle between the velocity of the charge and the magnetic field. Since the charge is stationary, $v = 0 \text{ m/s}$ and the magnetic force is zero.

When a moving charge is placed in an electric field, it experiences an electric force that is given by Equation 18.2. It does not matter whether the charge is stationary or moving.
When a charge moves \((v \neq 0 \text{ m/s})\) and its velocity is perpendicular to the magnetic field \((\theta = 90^\circ)\), it experiences a magnetic force, as specified by Equation 21.1.

**SOLUTION**

a. The magnitude of the electric force is \(F = q|E|\), where the magnitude of the electric field is related to the intensity \(S\) of the laser beam by \(S = c\varepsilon_0 E^2\) (Equation 24.5b). Therefore, the magnitude of the electric force is

\[
F = q|E| = |q|\sqrt{\frac{S}{c\varepsilon_0}} = \left(2.6 \times 10^{-8} \text{ C}\right) \sqrt{\frac{2.5 \times 10^3 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s} \left(8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)\right)}} = 2.5 \times 10^{-5} \text{ N}
\]

b. Since the particle is not moving, the magnetic force on it is zero, \(F = 0 \text{ N}\).

c. The electric force on the particle is the same whether it is moving or not, so the answer is the same as in part (a); \(F = 2.5 \times 10^{-5} \text{ N}\).

d. The magnitude of the magnetic force is given by Equation 21.1 as \(F = |q|vB\sin \theta\). The magnitude \(B\) of the magnetic field is related to the intensity \(S\) of the laser beam by \(S = cB^2/\mu_0\) (Equation 24.5c). Thus, the magnetic force is

\[
F = |q|vB\sin \theta = |q|v\sqrt{\frac{\mu_0 S}{c}} \sin \theta = \left(2.6 \times 10^{-8} \text{ C}\right)(3.7 \times 10^4 \text{ m/s}) \sqrt{\frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{3.00 \times 10^8 \text{ m/s}}} \frac{(2.5 \times 10^3 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}} \sin 90.0^\circ = 3.1 \times 10^{-3} \text{ N}
\]

31. **REASONING AND SOLUTION** The sun radiates sunlight (electromagnetic waves) uniformly in all directions, so the intensity at a distance \(r\) from the sun is given by Equation 16.9 as \(S = P/(4\pi r^2)\), where \(P\) is the power radiated by the sun. The power that strikes an area \(A_\perp\) oriented perpendicular to the direction in which the sunlight is radiated is \(P' = SA_\perp\), according to Equation 16.8. The 0.75-m\(^2\) patch of flat land on the equator at point \(Q\) is not perpendicular to the direction of the sunlight, however.
The figure at the right shows that
\[ A_\perp = (0.75 \text{ m}^2) \cos 27^\circ \]

Therefore, the power striking the patch of land is
\[ P' = S A_\perp = \left( \frac{P}{4\pi r^2} \right) (0.75 \text{ m}^2) \cos 27^\circ \]
\[ = \left[ \frac{3.9 \times 10^{26} \text{ W}}{4\pi (1.5 \times 10^{11} \text{ m})^2} \right] (0.75 \text{ m}^2) \cos 27^\circ = 920 \text{ W} \]

32. **REASONING** The average power \( \bar{P} \) delivered to the wall is equal to the energy delivered divided by the time \( t \). Thus, the time is equal to the energy divided by the average power, or \( t = \frac{\text{Energy}}{\bar{P}} \). The energy is given. The average power is equal to the average intensity \( \bar{S} \) times the area \( A \), so \( \bar{P} = \bar{S} A \). The area is known, and the average intensity is related to the rms-value \( B_{\text{rms}} \) of the wave’s magnetic field by \( \bar{S} = cB_{\text{rms}}^2 / \mu_0 \).

**SOLUTION** The time required to deliver the energy to the wall is
\[ t = \frac{\text{Energy}}{\bar{P}} \quad (6.10b) \]

Since \( \bar{P} = \bar{S} A \) (Equation 16.8), the expression for the time can be written as
\[ t = \frac{\text{Energy}}{\bar{P}} = \frac{\text{Energy}}{\bar{S} A} \quad (1) \]

The average intensity is related to \( B_{\text{rms}} \) by \( \bar{S} = cB_{\text{rms}}^2 / \mu_0 \) (Equation 24.5c), where \( c \) is the speed of light in a vacuum and \( \mu_0 \) is the permeability of free space. Substituting this expression into Equation (1) and noting that \( A = 1.30 \text{ cm}^2 = 1.30 \times 10^{-4} \text{ m}^2 \) gives
\[ t = \frac{\text{Energy}}{\bar{S} A} = \frac{\text{Energy}}{cB_{\text{rms}}^2 / \mu_0 \cdot A} \]
\[ = \frac{1850 \text{ J}}{(3.00 \times 10^8 \text{ m/s})(6.80 \times 10^{-4} \text{ T})^2 / (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \cdot (1.30 \times 10^{-4} \text{ m}^2)} = 0.129 \text{ s} \]
33. **REASONING** According to Equation 24.5b, the average intensity $\bar{S}$ of the infrared radiation is related to the rms value of the electric field $E_{\text{rms}}$ by $\bar{S} = c\varepsilon_0 E_{\text{rms}}^2$. According to Equation 16.8, the average power $\bar{P}$ is equal to the average intensity times the area $A$ to which the power is being delivered, the area being that of a circle or $A = \pi r^2$. Thus, $\bar{P} = \bar{S}A = \bar{S} (\pi r^2)$. The average power is given by Equation 6.10b as the energy $Q$ absorbed by the leg divided by the time $t$, so that $t = Q/\bar{P}$. The energy absorbed by the leg is related to the rise in temperature $\Delta T$ by Equation 12.4 as $Q = cm \Delta T$, where $c$ is the specific heat capacity and $m$ is the mass.

**SOLUTION**

a. The average intensity of the infrared radiation is

$$\bar{S} = c\varepsilon_0 E_{\text{rms}}^2$$  \hspace{1cm} (24.5b)

$$= (3.0 \times 10^8 \text{ m/s})
\left[8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)\right]
(2800 \text{ N/C})^2
= 2.1 \times 10^4 \text{ W/m}^2$$

b. The average power delivered to the leg is

$$\bar{P} = \bar{S}A = \bar{S}(\pi r^2) = (2.1 \times 10^4 \text{ W/m}^2)\pi (4.0 \times 10^{-2} \text{ m})^2 = 1.1 \times 10^2 \text{ W}$$ \hspace{1cm} (16.8)

c. Combining the relations $t = Q/\bar{P}$ and $Q = cm \Delta T$, the time required to raise the temperature by 2.0 $\text{C}^\circ$ is

$$t = \frac{Q}{\bar{P}} = \frac{cm \Delta T}{\bar{P}} = \frac{3500 \text{ J/(kg} \cdot \text{C}^\circ)(0.28 \text{ kg})(2.0 \text{ C}^\circ)}{1.1 \times 10^2 \text{ W}} = 18 \text{ s}$$

34. **REASONING** The rms magnetic field $B_{\text{rms}}$ and the average intensity $\bar{S}_1$ of the gamma radiation emitted from the surface of the magnetar are related by

$$\bar{S}_1 = \frac{c}{\mu_0} B_{\text{rms}}^2$$ \hspace{1cm} (24.5c)

where $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space and $c$ is the speed of light in a vacuum. The average intensity $\bar{S}_1$ of the gamma radiation is defined as the average power that passes perpendicularly through an area $A_1$. Therefore, if the radius of the magnetar is $r_1$, then the average intensity must be

$$\bar{S}_1 = \frac{\bar{P}}{A_1} = \frac{\bar{P}}{\pi r_1^2}$$ \hspace{1cm} (1)

In Equation (1), $A_1 = 4\pi r_1^2$ is the surface area of the magnetar (assumed to be spherical). When the pulse of gamma radiation reaches earth, the average power $\bar{P}_1$ is spread out
uniformly over the surface of a spherical surface of radius $R$ equal to the distance between earth and the magnetar. The area $A$ of this sphere is $A = 4\pi R^2$, and the average intensity $\bar{S}_2$ of the pulse when it reaches the telescope is given by $\bar{S}_2 = \frac{\bar{P}_1}{A}$, so we have that

$$\bar{P}_1 = A\bar{S}_2 = 4\pi R^2 \bar{S}_2$$

(2)

The average power $\bar{P}_2$ intercepted by the telescope detector is proportional to the average intensity $\bar{S}_2$ and the surface area $A_2$ of the detector: $\bar{P}_2 = A_2 \bar{S}_2$. Rearranging this relation, we obtain

$$\bar{S}_2 = \frac{\bar{P}_2}{A_2}$$

(3)

We will determine the average power $\bar{P}_2$ from the energy delivered to the detector and the duration $\Delta t$ of the pulse via Equation 6.10b:

$$\bar{P}_2 = \frac{\text{Energy}}{\Delta t}$$

(6.10b)

**SOLUTION** Solving Equation 24.5c for $B_{\text{rms}}$, we obtain

$$B_{\text{rms}}^2 = \frac{\mu_0 \bar{S}_1}{c} \quad \text{or} \quad B_{\text{rms}} = \sqrt{\frac{\mu_0 \bar{S}_1}{c}}$$

(4)

Substituting Equation (1) into Equation (4) yields

$$B_{\text{rms}} = \sqrt{\frac{\mu_0 \bar{P}_1}{4\pi c r_1^2}}$$

(5)

Substituting Equation (2) into Equation (5), we find that

$$B_{\text{rms}} = \sqrt{\frac{\mu_0 \bar{P}_1}{4\pi c r_1^2}} = \sqrt{\frac{\mu_0}{4\pi c r_1^2}} (4\pi R^2 \bar{S}_2) = \frac{R}{r_1} \sqrt{\frac{\mu_0 \bar{S}_2}{c}}$$

(6)

Substituting Equation (3) into Equation (6) gives

$$B_{\text{rms}} = \frac{R}{r_1} \sqrt{\frac{\mu_0 \bar{S}_2}{c}} = \frac{R}{r_1} \sqrt{\frac{\bar{P}_2}{c A_2}}$$

(7)

Replacing the average power $\bar{P}_2$ in Equation (7) with Equation 6.10b, we finally obtain

$$B_{\text{rms}} = \frac{R}{r_1} \sqrt{\frac{\mu_0 \text{Energy}}{c A_2 \Delta t}} = \frac{4.5 \times 10^{20} \text{ m}}{9.0 \times 10^{3} \text{ m}} \sqrt{\left(\frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{(8.4 \times 10^{-6} \text{ J})}\left(\frac{3.00 \times 10^8 \text{ m/s}}{(75 \text{ m}^2)}\right)(0.24 \text{ s})}\right) = 2.2 \times 10^6 \text{ T}$$
35. **REASONING** The Doppler effect for electromagnetic radiation is given by Equation 24.6:

\[ f_o = f_s \left(1 \pm \frac{v_{rel}}{c}\right) \quad \text{if} \quad v_{rel} << c \]

where \( f_o \) is the observed frequency, \( f_s \) is the frequency emitted by the source, and \( v_{rel} \) is the speed of the source relative to the observer. As discussed in the text, the plus sign applies when the source and the observer are moving toward one another, while the minus sign applies when they are moving apart. According to Equation 16.1, the wavelength of these waves is \( \lambda = c/f \). Therefore, the Doppler shift can be written in terms of wavelengths:

\[ \frac{1}{\lambda_o} = \frac{1}{\lambda_s} \left(1 \pm \frac{v_{rel}}{c}\right) \quad \text{if} \quad v_{rel} << c \]

**SOLUTION**

a. The wavelength \( \lambda_o \) of the light observed on earth is greater than the wavelength \( \lambda_s \) of the light when it is emitted from the distant galaxy (the source). Therefore, the frequency of the light observed on earth is less than the frequency of the light when it is emitted from the distant galaxy. Thus, the quantity in the brackets in Equation 24.6 must be less than one; it must be equal to \( 1 - (v_{rel} / c) \). Since the minus sign applies, we can conclude that **the galaxy must be receding from the earth**.

b. We can find the speed of the galaxy relative to the earth by solving the wavelength version of Equation 24.6 for \( v_{rel} \):

\[ v_{rel} = c \left(1 - \frac{\lambda_s}{\lambda_o}\right) = \left(3.0 \times 10^8 \text{ m/s}\right) \left(1 - \frac{434.1 \text{ nm}}{438.6 \text{ nm}}\right) = 3.1 \times 10^6 \text{ m/s} \]

36. **REASONING** Using the Doppler effect, we will find the relative speed between the speeding car and the police car. Since we know the speed of the police car relative to the ground, we can determine the speed of the car relative to the ground, once the relative speed \( v_{rel} \) is found.

There are two Doppler frequency changes in this situation. First, the speeder's car observes the wave frequency coming from the radar gun to have a frequency \( f_o \) that is different from the emitted frequency \( f_s \). The second Doppler shift occurs after the wave reflects from the speeder's car and returns to the police car.

The Doppler frequency for electromagnetic radiation is given by \( f_o = f_s \left[1 \pm (v_{rel} / c)\right] \) (Equation 24.6), where \( v_{rel} \) is the relative speed between the source and the observer of the radiation, and the plus sign applies when the source and the observer are moving toward one another, while the minus sign applies when they are moving apart. Since the distance
between the police car and the speeder's car is increasing, they are moving apart, and according to Equation 24.6, the first Doppler frequency change is given by $f_o - f_s = -f_s\left(\frac{v_{rel}}{c}\right)$. After the wave reflects from the speeder's car it returns to the police car where it is observed to have a frequency $f_o'$ that is different from its frequency $f_o$ at the instant of reflection. Equation 24.6 may again be used, this time to determine the second Doppler frequency shift: $f_o' - f_o = -f_o\left(\frac{v_{rel}}{c}\right)$. We can use these two equations for the frequency shifts to determine an expression for the total Doppler change in frequency. Adding the two equations, we have

$$(f_o' - f_o) + (f_o - f_s) = -f_o\left(\frac{v_{rel}}{c}\right) - f_s\left(\frac{v_{rel}}{c}\right)$$

$$f_o' - f_s = -\left[f_o\left(\frac{v_{rel}}{c}\right) + f_s\left(\frac{v_{rel}}{c}\right)\right] \approx -2f_s\left(\frac{v_{rel}}{c}\right)$$

where we have assumed that $f_s$ and $f_o$ differ only by a negligibly small amount, so that $f_o \approx f_s$. Rearranging, we have

$$f_s - f_o' \approx 2f_s\left(\frac{v_{rel}}{c}\right)$$

**SOLUTION** Solving for the relative speed $v_{rel}$ gives

$$v_{rel} \approx \left(\frac{f_s - f_o'}{2f_s}\right)c = \left[\frac{320 \text{ Hz}}{2(7.0 \times 10^9 \text{ Hz})}\right](3.0 \times 10^8 \text{ m/s}) = 6.9 \text{ m/s}$$

The relative speed $v_{rel}$ is related to the speeds of the vehicles with respect to the ground by $v_{rel} = v_{speeder} - v_{police}$. Therefore, the speeder's speed with respect to the ground is

$$v_{speeder} = v_{rel} + v_{police} = 6.9 \text{ m/s} + 25 \text{ m/s} = 32 \text{ m/s}$$

37. **REASONING** The Doppler effect for electromagnetic radiation is given by Equation 24.6, $f_o = f_s\left(1 \pm \frac{v_{rel}}{c}\right)$, where $f_o$ is the observed frequency, $f_s$ is the frequency emitted by the source, and $v_{rel}$ is the speed of the source relative to the observer. As discussed in the text, the plus sign applies when the source and the observer are moving toward one another, while the minus sign applies when they are moving apart.

**SOLUTION**
a. At location A, the galaxy is moving away from the earth with a relative speed of

$$v_{rel} = (1.6 \times 10^6 \text{ m/s}) - (0.4 \times 10^6 \text{ m/s}) = 1.2 \times 10^6 \text{ m/s}$$
Therefore, the minus sign in Equation 24.6 applies and the observed frequency for the light from region A is

\[ f_o = f_s \left( 1 - \frac{v_{rel}}{c} \right) = (6.200 \times 10^{14} \text{ Hz}) \left( 1 - \frac{1.2 \times 10^6 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} \right) = 6.175 \times 10^{14} \text{ Hz} \]

b. Similarly, at location B, the galaxy is moving away from the earth with a relative speed of

\[ v_{rel} = (1.6 \times 10^6 \text{ m/s}) + (0.4 \times 10^6 \text{ m/s}) = 2.0 \times 10^6 \text{ m/s} \]

The observed frequency for the light from region B is

\[ f_o = f_s \left( 1 - \frac{v_{rel}}{c} \right) = (6.200 \times 10^{14} \text{ Hz}) \left( 1 - \frac{2.0 \times 10^6 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} \right) = 6.159 \times 10^{14} \text{ Hz} \]

38. **REASONING** The observed frequency is \( f_o = f_s \left( 1 \pm \frac{v_{rel}}{c} \right) \) according to Equation 24.6. The frequency \( f_s \) emitted by the source is the same in each case, so that only the direction of the relative motion and the relative speed \( v_{rel} \) determine the observed frequency. In situations A and B the observer and the source move away from each other, and the minus sign in Equation 24.6 applies. In situation C the observer and the source move toward each other, and the plus sign applies. Thus, the observed frequency is largest in C. To distinguish between A and B, we note that the relative speed in A is \( 2v - v = v \), whereas in B the relative speed is \( 2v + v = 3v \). The greater relative speed means that the term \( v_{rel}/c \) is greater in B than in A, and since the minus sign applies, the observed frequency is more reduced in B than in A. We conclude, then, that the situations are ranked in descending order according to the observed frequencies as follows: C (largest), A, B

**SOLUTION** To apply Equation 24.6 to calculate the observed frequencies, we need the relative speed in each situation. The relative speeds and the observed frequencies are:

**[Situation A, minus sign in Equation 24.6]**

\[ v_{rel} = 2v - v = v \]

\[ f_o = f_s \left( 1 - \frac{v_{rel}}{c} \right) = f_s \left( 1 - \frac{v}{c} \right) = (4.57 \times 10^{14} \text{ Hz}) \left( 1 - \frac{1.50 \times 10^6 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} \right) = 4.55 \times 10^{14} \text{ Hz} \]

**[Situation B, minus sign in Equation 24.6]**

\[ v_{rel} = 2v + v = 3v \]

\[ f_o = f_s \left( 1 - \frac{v_{rel}}{c} \right) = f_s \left( 1 - \frac{3v}{c} \right) = (4.57 \times 10^{14} \text{ Hz}) \left[ 1 - \frac{3(1.50 \times 10^6 \text{ m/s})}{3.00 \times 10^8 \text{ m/s}} \right] = 4.50 \times 10^{14} \text{ Hz} \]
[Situation C, plus sign in Equation 24.6]

\[ v_{rel} = v + v = 2v \]

\[ f_0 = f_s \left(1 + \frac{v_{rel}}{c}\right) = f_s \left(1 + \frac{2v}{c}\right) = \left(4.57 \times 10^{14} \text{ Hz}\right) \left[1 + \frac{2(1.50 \times 10^6 \text{ m/s})}{3.00 \times 10^8 \text{ m/s}}\right] = 4.62 \times 10^{14} \text{ Hz} \]

39. **REASONING AND SOLUTION**

a. The polarizer reduces the intensity of the light by a factor of two or to \(0.55 \text{ W/m}^2\).

b. The intensity of the light leaving the analyzer is given by Malus’ law.

\[ S = (0.55 \text{ W/m}^2) \cos^2 75^\circ = 3.7 \times 10^{-2} \text{ W/m}^2 \]

40. **REASONING**

**Drawing A** The transmission axes of the polarizer and analyzer are parallel to each other, so all the light transmitted by the polarizer is completely transmitted by the analyzer.

**Drawing B** The transmission axes of the polarizer and analyzer are perpendicular to each other, so no light is transmitted through the analyzer.

**Drawing C** The transmission axes of the polarizer and analyzer make an angle of 30.0° with respect to each other. Thus, some of the light transmitted by the polarizer, but not all, is transmitted through the analyzer.

Therefore, we expect the transmitted intensities to be in the following decreasing order (largest first): A, C, B.

**SOLUTION** Since the incident light is unpolarized, the average intensity \(\bar{S}_1\) of the light transmitted by the polarizer is one-half the average intensity \(\bar{S}_0\) of the incident light, or \(\bar{S}_1 = \frac{1}{2} \bar{S}_0 = \frac{1}{2} \left(48 \text{ W/m}^2\right) = 24 \text{ W/m}^2\). The average intensity \(\bar{S}_2\) of the light transmitted by the analyzer is given by Malus’ law, Equation 24.7, as \(\bar{S}_2 = \bar{S}_1 \cos^2 \theta\), where \(\theta\) is the angle between the direction of polarization and the transmission axis. The average intensity of the transmitted beams for each of the three cases is

A \[ \bar{S}_2 = \bar{S}_1 \cos^2 \theta = (24 \text{ W/m}^2) \cos^2 0^\circ = 24 \text{ W/m}^2 \]

B \[ \bar{S}_2 = \bar{S}_1 \cos^2 \theta = (24 \text{ W/m}^2) \cos^2 90^\circ = 0 \text{ W/m}^2 \]

C \[ \bar{S}_2 = \bar{S}_1 \cos^2 \theta = (24 \text{ W/m}^2) \cos^2 (60.0^\circ - 30.0^\circ) = 18 \text{ W/m}^2 \]
41. **REASONING** Malus’ law, $S = S_0 \cos^2 \theta$ (Equation 24.7), relates the average intensity $S_0$ of polarized light incident on the polarizing sheet to the average intensity $S$ of light transmitted by the sheet, where $\theta$ is the angle between the polarization axis of the incident light and the transmission axis of the polarizing sheet. The incident light is horizontally polarized, so the angle $\theta$ is measured from the horizontal, and is, therefore, the angle we seek.

**SOLUTION** Solving $S = S_0 \cos^2 \theta$ (Equation 24.7) for $\theta$, we obtain

$$\cos^2 \theta = \frac{S}{S_0} \quad \text{or} \quad \cos \theta = \sqrt{\frac{S}{S_0}} \quad \text{or} \quad \theta = \cos^{-1} \left( \sqrt{\frac{S}{S_0}} \right) \quad (2)$$

Substituting the given values of the average incident and transmitted intensities yields

$$\theta = \cos^{-1} \left( \sqrt{\frac{0.764 \text{ W/m}^2}{0.883 \text{ W/m}^2}} \right) = 21.5^\circ$$

42. **REASONING** Since no light passes through the second sheet, it must be in a crossed configuration with the first sheet. In other words, its transmission axis must be perpendicular to the transmission axis of the first sheet. Therefore, to find its orientation with respect to the vertical, we will determine the angle that the axis of the first sheet makes with respect to the vertical and add $90.0^\circ$ to it. The angle that the axis of the first sheet makes with respect to the vertical can be obtained directly from Malus’ law.

**SOLUTION** The angle $\theta_2$ that the second sheet makes with respect to the vertical is

$$\theta_2 = \theta_1 + 90.0^\circ$$

where $\theta_1$ is the angle giving the orientation of the axis of the first sheet. Using Malus’ law (Equation 24.7) and recognizing that the ratio of the intensity $S$ passing through the first sheet to the incident intensity $S_0$ is $S / S_0 = 0.94$, we find that

$$S = S_0 \cos^2 \theta_1 \quad \text{or} \quad \frac{S}{S_0} = 0.94 = \cos^2 \theta_1 \quad \text{or} \quad \theta_1 = \cos^{-1} \sqrt{0.94} = 14^\circ$$

The angle of the axis of the second sheet with respect to the vertical is, then,

$$\theta_2 = \theta_1 + 90.0^\circ = 14^\circ + 90.0^\circ = 104^\circ$$
43. **REASONING** If the intensity of the unpolarized light is $I_0$, the intensity of the polarized light leaving the polarizer is $\frac{1}{2}I_0$. By Malus’ law, the intensity of the light leaving the insert is $\frac{1}{2}I_0 \cos^2 \theta$. From the results of Conceptual Example 8, the intensity of light leaving the analyzer is $\frac{1}{2}I_0 \cos^2 \theta \sin^2 \theta$.

**SOLUTION** The intensity $I$ of light that reaches the photocell is

$$I = \frac{1}{2}I_0 \cos^2 \theta \sin^2 \theta = \frac{1}{2}(150 \text{ W/m}^2) \cos^2 30.0^\circ \sin^2 30.0^\circ = 14 \text{ W/m}^2$$

44. **REASONING** If the incident light is unpolarized, the average intensity of the transmitted light is one-half the average intensity of the incident light, independent of the angle of the transmission axis. Thus, the average intensity of the transmitted light remains the same as the polarizing material is rotated.

If the incident light is polarized along the $z$ axis, the direction of polarization and the transmission axis are initially parallel to each other, and the maximum amount of light is transmitted. As the polarizing material is rotated, the intensity of the transmitted light decreases in accord with Malus’ law.

If the incident light is polarized along the $y$ axis, the direction of polarization and the transmission axis are initially perpendicular to each other, and no light is transmitted. As the polarizing material is rotated, the average intensity of the transmitted light increases.

**SOLUTION**

a. Since the incident light is unpolarized, the average intensity of the transmitted light is one-half the average intensity of the incident light. Therefore, for both $\alpha = 0^\circ$ and $35^\circ$, we have

$$\bar{S} = \frac{1}{2}\bar{S}_0 = \frac{1}{2}(7.0 \text{ W/m}^2) = 3.5 \text{ W/m}^2$$

b. When the incident light is polarized along the $z$ axis, the direction of polarization and the transmission axis are initially parallel to each other. Therefore, the angle $\alpha$ is the same as the angle $\theta$ between the transmission axis of the polarizer and the direction of the polarization. According to Malus’ law (Equation 24.7), the average intensity of the transmitted light is given by

$$\bar{S} = \bar{S}_0 \cos^2 \theta = (7.0 \text{ W/m}^2) \cos^2 0^\circ = 7.0 \text{ W/m}^2$$

$$\bar{S} = \bar{S}_0 \cos^2 \theta = (7.0 \text{ W/m}^2) \cos^2 35^\circ = 4.7 \text{ W/m}^2$$
c. When the incident light is polarized along the y axis, the direction of polarization and the transmission axis are initially perpendicular to each other. The angle $\theta$ in Malus’ law is the angle between the direction of polarization (along the y axis) and the transmission axis (measured relative to the z axis). It is related to the angle $\alpha$ according to $\theta = 90.0^\circ - \alpha$. The average intensity of the transmitted light is, therefore,

$$\overline{S} = \overline{S}_0 \cos^2 \theta = (7.0 \text{ W/m}^2) \cos^2 (90.0^\circ - 0^\circ) = 0 \text{ W/m}^2$$

$$\overline{S} = \overline{S}_0 \cos^2 \theta = (7.0 \text{ W/m}^2) \cos^2 (90.0^\circ - 35^\circ) = 2.3 \text{ W/m}^2$$

The table below summarizes the results:

<table>
<thead>
<tr>
<th>Incident Light</th>
<th>Average Intensity of Transmitted Light</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha = 0^\circ$</td>
</tr>
<tr>
<td>(a) Unpolarized</td>
<td>3.5 W/m$^2$</td>
</tr>
<tr>
<td>(b) Polarized parallel</td>
<td>7.0 W/m$^2$</td>
</tr>
<tr>
<td>to z axis</td>
<td></td>
</tr>
<tr>
<td>(c) Polarized parallel</td>
<td>0 W/m$^2$</td>
</tr>
<tr>
<td>to y axis</td>
<td></td>
</tr>
</tbody>
</table>

45. **REASONING** Since the incident beam is unpolarized, the intensity of the light transmitted by the first sheet of polarizing material is one-half the intensity of the incident beam. The beams striking the second and third sheets of polarizing material are polarized, so the average intensity $\overline{S}$ of the light transmitted by each sheet is given by Malus’ law, $\overline{S} = \overline{S}_0 \cos^2 \theta$, where $\overline{S}_0$ is the average intensity of the light incident on each sheet.

**SOLUTION** The average intensity $\overline{S}_1$ of the light leaving the first sheet is one-half the intensity of the incident beam, so $\overline{S}_1 = \frac{1}{2} (1260.0 \text{ W/m}^2) = 630.0 \text{ W/m}^2$. The intensity $\overline{S}_2$ of the light leaving the second sheet of polarizing material is given by Malus’ law, Equation 24.7, $\overline{S}_2 = \overline{S}_1 \cos^2 \theta$, where $\theta$ is the angle between the polarization of the incident beam and the transmission axis of the second sheet:

$$\overline{S}_2 = (630.0 \text{ W/m}^2) \cos^2 (55.0^\circ - 19.0^\circ) = 412 \text{ W/m}^2$$

The intensity $\overline{S}_3$ of the light leaving the third sheet of polarizing material is $\overline{S}_3 = \overline{S}_2 \cos^2 \theta$, where $\theta$ is the angle between the polarization of the incident beam and the transmission axis of the third sheet:

$$\overline{S}_3 = (412 \text{ W/m}^2) \cos^2 (100.0^\circ - 55.0^\circ) = 206 \text{ W/m}^2$$
46. **REASONING** The polarizer, the insert and the analyzer in the set-up in Figure 24.24a all reduce the intensity of the light that reaches the photocell. The polarizer reduces the intensity by a factor of one-half, as described in Section 24.6 of the text; if the average intensity of the incident unpolarized light is $\bar{I}$, the average intensity of the polarized light that leaves the polarizer and strikes the insert is $\bar{S}_0 = \bar{I} / 2$. According to Malus' law (see Equation 24.7), the average intensity $\bar{S}_{\text{insert}}$ of the light leaving the insert is $\bar{S}_{\text{insert}} = \bar{S}_0 \cos^2 \theta$, where $\theta$ is the relative angle between the transmission axes of the polarizer and the insert. The intensity of the light is further reduced as the polarized light passes through the analyzer. Malus' law can be used in succession at each piece of polarizing material to determine the intensity that reaches the photocell, both with and without the presence of the analyzer.

**SOLUTION** When the analyzer is present, the average intensity reaching the photocell is equal to the average intensity that leaves the analyzer. The average intensity leaving the analyzer is, according to Malus' law, $\bar{S}_{\text{insert}} \cos^2 \phi$, where $\bar{S}_{\text{insert}}$ is the average intensity that leaves the insert and reaches the analyzer, and $\phi$ is the relative angle between the transmission axes of the analyzer and insert. From Figure 24.24a we see that $\phi = 90^\circ - \theta$. The average intensity of the light leaving the insert is $\bar{S}_{\text{insert}} = \bar{S}_0 \cos^2 \theta$, according to Malus' law. Therefore, when the analyzer is present as shown in Figure 24.24a, the average intensity leaving the analyzer and reaching the photocell is

$$\bar{S}_{\text{photocell}} = \bar{S}_{\text{insert}} \cos^2 \phi = \left( \bar{S}_0 \cos^2 \theta \right) \cos^2 (90^\circ - \theta)$$

This expression can be solved for $\bar{S}_0$ to determine the average intensity leaving the polarizer:

$$\bar{S}_0 = \frac{\bar{S}_{\text{photocell}}}{(\cos^2 \theta) \cos^2 (90^\circ - \theta)} = \frac{110 \text{ W/m}^2}{(\cos^2 23^\circ) \cos^2 (90^\circ - 23^\circ)} = 850 \text{ W/m}^2$$

If the analyzer were removed from the setup, everything else remaining the same, the intensity reaching the photocell would be equal to the intensity that leaves the insert. Therefore, if the analyzer were removed, the intensity reaching the photocell would be

$$\bar{S}_{\text{photocell}} = \bar{S}_{\text{insert}} = \bar{S}_0 \cos^2 \theta = (850 \text{ W/m}^2) \cos^2 23^\circ = 720 \text{ W/m}^2$$

47. **SSM REASONING** The average intensity of light leaving each analyzer is given by Malus' Law (Equation 24.7). Thus, intensity of the light transmitted through the first analyzer is

$$\bar{S}_1 = \bar{S}_0 \cos^2 27^\circ$$

Similarly, the intensity of the light transmitted through the second analyzer is

$$\bar{S}_2 = \bar{S}_1 \cos^2 27^\circ = \bar{S}_0 \cos^4 27^\circ$$
And the intensity of the light transmitted through the third analyzer is

\[ \bar{S}_3 = \bar{S}_2 \cos^2 27^\circ = \bar{S}_0 \cos^6 27^\circ \]

If we generalize for the \( N \)th analyzer, we deduce that

\[ \bar{S}_N = \bar{S}_{N-1} \cos^2 27^\circ = \bar{S}_0 \cos^{2N} 27^\circ \]

Since we want the light reaching the photocell to have an intensity that is reduced by at least a factor of one hundred relative to the first analyzer, we want \( \frac{\bar{S}_N}{\bar{S}_0} = 0.010 \). Therefore, we need to find \( N \) such that \( \cos^{2N} 27^\circ = 0.010 \). This expression can be solved for \( N \).

**SOLUTION** Taking the common logarithm of both sides of the last expression gives

\[ 2N \log(\cos 27^\circ) = \log 0.010 \quad \text{or} \quad N = \frac{\log 0.010}{2 \log(\cos 27^\circ)} = 20 \]

48. **REASONING** No light intensity will pass through two adjacent polarizers that are in a crossed configuration, that is, whose transmission axes are oriented perpendicular to one another. With this in mind, let's remove the polarizers one by one (see the following drawing). When A is removed, no two of the remaining adjacent polarizers are crossed. When B is removed, A and C are left in a crossed configuration. When C is removed, B and D are left in a crossed configuration. When D is removed, no two of the remaining adjacent polarizers are crossed. Thus, when sheet B or C is removed, the intensity transmitted on the right is zero, and when sheet A or D is removed, the intensity transmitted on the right is greater than zero.

We can anticipate that the greater intensity is transmitted on the right when sheet D is removed. To begin with, we note that the polarization directions of the light striking B and C are the same, no matter whether A or D is removed, with the result that B and C absorb the same fraction of the intensity in either situation. When A is removed, however, D is a third sheet that absorbs light intensity. In contrast, when D is removed, A is present as a third sheet, but it absorbs none of the light intensity. This is because the transmission axis of A is vertical and matches the direction in which the incident light is polarized. We conclude, therefore, that the greater light intensity is transmitted when D is removed.
**SOLUTION** When light with an average intensity $\bar{S}_0$ is polarized at an angle $\theta$ with respect to the polarization axis of a polarizer, the average intensity $\bar{S}$ that is transmitted through the polarizer is given by Malus’ law as $\bar{S} = \bar{S}_0 \cos^2 \theta$ (Equation 24.7). The light that passes through is polarized in the direction of the transmission axis. In this problem, each of the polarizers, therefore, transmits a light intensity that is smaller than the incident light by a factor of $\cos^2 \theta$. We use this insight now to determine the transmitted light intensity in the two situations that result when A is removed and when D is removed.

[A is removed; B, C, and D remain]

$$\bar{S} = \left(27 \text{ W/m}^2\right) \cos^2 30.0^\circ \cos^2 60.0^\circ \cos^2 30.0^\circ = 3.8 \text{ W/m}^2$$

due to B \hspace{2cm} \text{due to C} \hspace{2cm} \text{due to D}

[B is removed; A, C, and D remain]

$$\bar{S} = 0 \text{ W/m}^2$$

[C is removed; A, B, and D remain]

$$\bar{S} = 0 \text{ W/m}^2$$

[D is removed; A B, and C remain]

$$\bar{S} = \left(27 \text{ W/m}^2\right) \cos^2 0.0^\circ \cos^2 30.0^\circ \cos^2 60.0^\circ = 5.1 \text{ W/m}^2$$

due to A \hspace{2cm} \text{due to B} \hspace{2cm} \text{due to C}

49. **REASONING** The wavelength $\lambda$ of a wave is related to its speed $v$ and frequency $f$ by $\lambda = \frac{v}{f}$ (Equation 16.1). Since blue light and orange light are electromagnetic waves, they travel through a vacuum at the speed of light $c$; thus, $v = c$.

**SOLUTION**

a. The wavelength of the blue light is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{6.34 \times 10^{14} \text{ Hz}} = 4.73 \times 10^{-7} \text{ m}$$

Since 1 nm = $10^{-9}$ m,

$$\lambda = (4.73 \times 10^{-7} \text{ m}) \left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = 473 \text{ nm}$$

b. In a similar manner, we find that the wavelength of the orange light is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{4.95 \times 10^{14} \text{ Hz}} = 6.06 \times 10^{-7} \text{ m} = 606 \text{ nm}$$
50. **REASONING** The relationship between the intensity $S$ of an electromagnetic wave and its electric field $E$ is given by Equation 24.5b as $S = c\varepsilon_0 E^2$. For example, if the magnitude of the electric field triples, the intensity increases by a factor of $3^2 = 9$.

The magnitude of the magnetic field is given by Equation 24.3 as $B = \frac{E}{c}$. Even though the magnitude of the magnetic field is much smaller than that of the electric field, tripling the magnetic field also causes the intensity to increase by a factor of $3^2 = 9$. This can be seen by examining Equation 24.5c, $S = \frac{c}{\mu_0} B^2$.

**SOLUTION**

a. When the magnitude of the electric field is 315 N/C, the intensity of the electromagnetic wave is

$$S = c\varepsilon_0 E^2 = \left(3.00 \times 10^8 \text{ m/s}\right) \left[8.85 \times 10^{-12} \text{ C}^2 / \left(\text{N} \cdot \text{m}^2\right)\right] (315 \text{ N/C})^2 = 263 \text{ W/m}^2$$

b. The magnitudes of the magnetic fields associated with each electric field are

$$B = \frac{E}{c} = \frac{315 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = 1.05 \times 10^{-6} \text{ T}$$

$$B = \frac{E}{c} = \frac{945 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = 3.15 \times 10^{-6} \text{ T}$$

c. The intensities of the waves associated with each value of the magnetic field are

$$S = \frac{c}{\mu_0} B^2 = \frac{3.00 \times 10^8 \text{ m/s}}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \left(1.05 \times 10^{-6} \text{ T}\right)^2 = 263 \text{ W/m}^2$$

$$S = \frac{c}{\mu_0} B^2 = \frac{3.00 \times 10^8 \text{ m/s}}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \left(3.15 \times 10^{-6} \text{ T}\right)^2 = 2370 \text{ W/m}^2$$

51. **SSM** **REASONING** The electromagnetic wave will be picked up by the radio when the resonant frequency $f_0$ of the circuit in Figure 24.4 is equal to the frequency of the broadcast wave, or $f_0 = 1400$ kHz. This frequency, in turn, is related to the capacitance $C$ and inductance $L$ of the circuit via $f_0 = \frac{1}{2\pi\sqrt{LC}}$ (Equation 23.10). Since $C$ is known, we can use this relation to find the inductance.
**SOLUTION** Solving the relation \( f_0 = 1/(2\pi\sqrt{LC}) \) for the inductance \( L \), we find that

\[
L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 \left(1400 \times 10^3 \text{ Hz}\right)^2 \left(8.4 \times 10^{-11} \text{ F}\right)} = 1.5 \times 10^{-4} \text{ H}
\]

---

52. **REASONING AND SOLUTION** The number of wavelengths that can fit across the width \( W \) of your thumb is \( W/\lambda \). From Equation 16.1, we know that \( \lambda = c/f \), so

\[
\text{No. of wavelengths} = \frac{W}{\lambda} = \frac{Wf}{c} = \frac{(2.0 \times 10^{-2} \text{ m})(5.5 \times 10^{14} \text{ Hz})}{3.0 \times 10^8 \text{ m/s}} = 3.7 \times 10^4
\]

---

53. **SSM REASONING AND SOLUTION**

a. According to Equation 24.5b, the average intensity is \( \bar{S} = c\varepsilon_0 E_{\text{rms}}^2 \). In addition, the average intensity is the average power \( \bar{P} \) divided by the area \( A \). Therefore,

\[
E_{\text{rms}} = \sqrt{\frac{\bar{S}}{c\varepsilon_0}} = \sqrt{\frac{\bar{P}}{c\varepsilon_0 A}} = \sqrt{\frac{1.20 \times 10^4 \text{ W}}{(3.00 \times 10^8 \text{ m/s})[8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)](135 \text{ m}^2)}} = 183 \text{ N/C}
\]

b. Then, from \( E_{\text{rms}} = cB_{\text{rms}} \) (Equation 24.3), we have

\[
B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{183 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = 6.10 \times 10^{-7} \text{ T}
\]

---

54. **REASONING** There are two Doppler frequency changes in the emitted wave in this case. First, the speeder's car observes the wave frequency coming from the radar gun to have a frequency \( f_0 \) that is different from the emitted (source) frequency \( f_s \). The wave then reflects and returns to the police car, where it is observed to have a frequency \( f'_0 \) that is different than its frequency \( f_s \) at the instant of reflection. Although the police car is now moving, the relative motion of the two vehicles is one of approach. In Example 6, it is shown that, when the source and the observer of the radar are approaching each other, the magnitude of the difference between frequency of the emitted wave and the wave that returns to the police car after reflecting from the speeder's car is

\[
f'_0 - f_s \approx 2 f_s \left(\frac{v_{\text{rel}}}{c}\right)
\]

where \( v_{\text{rel}} \) is the relative speed between the speeding car and the police car.
**SOLUTION** Since the police car is moving to the right at 27 m/s, while the speeder is coming from behind at 39 m/s, the relative speed \( v_{\text{rel}} \) is 39 m/s - 27 m/s = 12 m/s. The total Doppler change in frequency is, therefore,

\[
f'_o - f_s = 2(8.0 \times 10^9 \text{ Hz}) \left( \frac{12 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} \right) = 640 \text{ Hz}
\]

55. **SSM REASONING** In experiment 1, the light falling on the polarizer is unpolarized, so the average intensity \( \overline{S} \) transmitted by it is \( \overline{S} = \frac{1}{2} \overline{S}' \), where \( \overline{S}' \) is the intensity of the incident light. The intensity of the light passing through the analyzer and reaching the photocell can be found by using Malus' law.

In the second experiment, all of the polarized light passes through the polarizer, so the average intensity reaching the analyzer is \( \overline{S}' \). The intensity of the light passing through the analyzer and reaching the photocell can again be found by using Malus' law. By setting the intensities of the light reaching the photocells in experiments 1 and 2 equal, we can determine the number of additional degrees that the analyzer in experiment 2 must be rotated relative to that in experiment 1.

**SOLUTION** In experiment 1 the light intensity incident on the analyzer is \( \overline{S}_0 = \frac{1}{2} \overline{S}' \). Malus' law (Equation 24.7) gives the average intensity \( \overline{S}_1 \) of the light reaching the photocell as

\[
\overline{S}_1 = \overline{S}_0 \cos^2 60.0^\circ = \frac{1}{2} \overline{S}' \cos^2 60.0^\circ
\]

In experiment 2 the incident light is polarized along the axis of the polarizer, so all the light is transmitted by the polarizer. Thus, the light intensity incident on the analyzer is \( \overline{S}_0 = \overline{S}' \).

Malus' law again gives the average intensity \( \overline{S}_2 \) of the light reaching the photocell as

\[
\overline{S}_2 = \overline{S}_0 \cos^2 \theta = \overline{S}' \cos^2 \theta
\]

Setting \( \overline{S}_1 = \overline{S}_2 \), we have

\[
\frac{1}{2} \overline{S}' \cos^2 60.0^\circ = \overline{S}' \cos^2 \theta
\]

Algebraically eliminating \( \overline{S}' \) from this equation, we have that

\[
\frac{1}{2} \cos^2 60.0^\circ = \cos^2 \theta \quad \text{or} \quad \theta = \cos^{-1} \left( \sqrt{\frac{1}{2} \cos^2 60.0^\circ} \right) = 69.3^\circ
\]

The number of additional degrees that the analyzer must be rotated is 69.3° - 60.0° = 9.3°. The angle \( \theta \) is increased by the additional rotation.
56. \textbf{REASONING AND SOLUTION} The intensity $S$ of a wave is the power passing perpendicularly through a surface divided by the area $A$ of the surface. But power is the total energy $U$ per unit time $t$, so the intensity can be written as

\[ S = \frac{\text{Total energy}}{\text{Time} \cdot \text{Area}} = \frac{U}{tA} \]

Equation 24.5c relates the intensity $S$ of the electromagnetic wave to the magnitude $B$ of its magnetic field; namely $S = (c / \mu_0)B^2$. Combining these two results, we have

\[ \frac{U}{tA} = \frac{c}{\mu_0}B^2 \]

If the rms value for the magnetic field is used, the energy becomes the average energy $\bar{U}$. Thus, the average energy that this wave carries through the window in a 45 s phone call is

\[ \bar{U} = \frac{c}{\mu_0}B_{\text{rms}}^2 tA = \left( \frac{3.0 \times 10^8 \text{ m/s}}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \right) (1.5 \times 10^{-10} \text{ T})^2 (45 \text{ s})(0.20 \text{ m}^2) = 4.8 \times 10^{-5} \text{ J} \]

57. \textbf{REASONING} The equation that represents the wave mathematically is $y = A \sin(2\pi ft - 2\pi x / \lambda)$. In this expression the amplitude is $A = 156 \text{ N/C}$. The wavelength $\lambda$ can be calculated using Equation 16.1, and we obtain

\[ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50 \times 10^8 \text{ Hz}} = 2.00 \text{ m} \]

\textbf{SOLUTION}

a. For $t = 0$ s, the wave expression becomes

\[ y = A \sin(2\pi f t - 2\pi x / \lambda) = 156 \sin \left( \frac{2\pi f(0) - 2\pi x}{2.00} \right) = -156 \sin \left( \frac{2\pi x}{2.00} \right) = -156 \sin (\pi x) \]

In this result, the units are suppressed for convenience. The following table gives the values of the electric field obtained using this version of the wave expression with the given values of the position $x$. The term $\pi x$ is in radians when $x$ is in meters, and conversion from radians to degrees is accomplished using the fact that $2\pi \text{ rad} = 360^\circ$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = -156 \sin (\pi x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 m</td>
<td>$-156 \sin (0) = -156 \sin (0^\circ) = 0$</td>
</tr>
<tr>
<td>0.50 m</td>
<td>$-156 \sin (0.50 \pi) = -156 \sin (90^\circ) = -156$</td>
</tr>
<tr>
<td>1.00 m</td>
<td>$-156 \sin (1.00 \pi) = -156 \sin (180^\circ) = 0$</td>
</tr>
<tr>
<td>1.50 m</td>
<td>$-156 \sin (1.50 \pi) = -156 \sin (270^\circ) = +156$</td>
</tr>
<tr>
<td>2.00 m</td>
<td>$-156 \sin (2.00 \pi) = -156 \sin (360^\circ) = 0$</td>
</tr>
</tbody>
</table>
These values for the electric field are plotted in the graph shown at the right.

b. For $t = T/4$, we use the fact that $f = 1/T$, and the wave expression becomes

$$y = A \sin \left(2\pi ft - 2\pi x / \lambda \right) = 156 \sin \left[2\pi \left(\frac{1}{T}\right) \left(\frac{T}{4}\right) \frac{2\pi x}{2.00}\right] = 156 \sin \left(\frac{\pi}{2} - \pi x\right) = 156 \cos (\pi x)$$

In this result, the units are suppressed for convenience. The following table gives the values of the electric field obtained using this version of the wave expression with the given values of the position $x$. The term $\pi x$ is in radians when $x$ is in meters, and conversion from radians to degrees is accomplished using the fact that $2\pi \text{ rad} = 360^\circ$.

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These values for the electric field are plotted in the graph shown at the right.

58. **REASONING** The rms value $E_{\text{rms}}$ of the electric field is related to the average intensity $\bar{S}$ of the light by $\bar{S} = c\varepsilon_0 E_{\text{rms}}^2$ (Equation 24.5b). The average intensity $\bar{S}$ of the light transmitted by the polarizer is related to the incident intensity $\bar{S}_0$ by Malus' law,
\[ S = \bar{S}_0 \cos^2 \theta, \] where \( \theta \) is the angle between the transmission axis and the direction of polarization. These two relations will allow us to determine the rms value of the electric field.

**SOLUTION** Combining the two equations given above and solving for the rms value of the electric field, we have

\[
E_{\text{rms}} = \sqrt{\frac{\bar{S}_0 \cos^2 \theta}{c \varepsilon_0}} = \sqrt{\frac{(15 \text{ W/m}^2) \cos^2 25^\circ}{(3.0 \times 10^8 \text{ m/s}) \left[ 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2) \right]}} = 68 \text{ N/C}
\]

59. **REASONING AND SOLUTION** The intensity of the laser light is \( S = \frac{P}{A} = cu \), where \( u \) is the energy density of the light. The energy in a section of length \( L \) of the cylindrical beam is \( U = uAL \) or

\[
U = \frac{PL}{c} = \frac{(0.750 \text{ W})(2.50 \text{ m})}{3.00 \times 10^8 \text{ m/s}} = 6.25 \times 10^{-9} \text{ J}
\]

60. **REASONING** The fraction of the sun’s power that is intercepted by Mercury is the power \( P_{\text{intercepted}} \) intercepted by Mercury divided by the power \( P_{\text{sun}} \) radiated by the sun, or Fraction = \( P_{\text{intercepted}}/P_{\text{sun}} \). According to Equation 16.8, the power intercepted by Mercury is the intensity \( S \) of the radiation at Mercury’s location times the area \( A \) that Mercury presents to the radiation. Since the sun radiates uniformly in all directions, we can use Equation 16.9 to evaluate the intensity at Mercury’s location. The planet Mercury looks like a circular disk to the radiation (just like the moon looks like a disk to anyone viewing it); this area is \( A = \pi r_{\text{Mercury}}^2 \), where \( r_{\text{Mercury}} \) is the radius of Mercury.

**SOLUTION** The fraction of the sun’s power that is intercepted by Mercury is

\[
\text{Fraction} = \frac{\text{Power intercepted by Mercury}}{\text{Power radiated by sun}} = \frac{P_{\text{intercepted}}}{P_{\text{sun}}}
\]

According to Equation 16.8, the power \( P_{\text{intercepted}} \) by Mercury is equal to the intensity \( S \) of the radiation at Mercury times the area \( A \) of the Mercury disk, or \( P_{\text{intercepted}} = SA \). Substituting this expression into Equation (1) gives

\[
\text{Fraction} = \frac{P_{\text{intercepted}}}{P_{\text{sun}}} = \frac{SA}{P_{\text{sun}}}
\]

Since the sun radiates uniformly in all directions, the intensity of the radiation at Mercury’s location is \( S = P_{\text{sun}}/4\pi r^2 \) (Equation 16.9), where \( r \) is the mean distance from the sun to
Mercury. As mentioned in the *REASONING* section, the area of the Mercury disk is 
\[ A = \pi r_{\text{Mercury}}^2 \]. Substituting these expressions for \( S \) and \( A \) into Equation (2) yields

\[
\text{Fraction} = \frac{S \cdot A}{P_{\text{sun}}} = \frac{P_{\text{sun}} / (\pi r_{\text{Mercury}}^2)}{P_{\text{sun}} / 4r^2} = \frac{r_{\text{Mercury}}^2}{4r^2} = \frac{(2.44 \times 10^6 \text{ m})^2}{4(5.79 \times 10^{10} \text{ m})^2} = 4.44 \times 10^{-10}
\]

61. **SSM REASONING** The electromagnetic solar power that strikes an area \( A_{\perp} \) oriented perpendicular to the direction in which the sunlight is radiated is \( P = S A_{\perp} \), where \( S \) is the intensity of the sunlight. In the problem, the solar panels are not oriented perpendicular to the direction of the sunlight, because it strikes the panels at an angle \( \theta \) with respect to the normal. We wish to find the solar power that impinges on the solar panels when \( \theta = 25^\circ \), given that the incident power is 2600 W when \( \theta = 65^\circ \).

**SOLUTION** When the angle that the sunlight makes with the normal to the solar panel is \( \theta \), the power that strikes the solar panel is given by \( P = S A \cos \theta \), where the area perpendicular to the sunlight is \( A_{\perp} = A \cos \theta \) (see the drawing). Therefore we can write

\[
\frac{P_2}{P_1} = \frac{S A \cos \theta_2}{S A \cos \theta_1}
\]

where the intensity \( S \) of the sunlight that reaches the panel, as well as the area \( A \), are the same in both cases. Therefore, we have

\[
\frac{P_2}{P_1} = \frac{\cos \theta_2}{\cos \theta_1}
\]

Solving for \( P_2 \), we find that when \( \theta_2 = 35^\circ \), the solar power impinging on the panel is

\[
P_2 = P_1 \left( \frac{\cos \theta_2}{\cos \theta_1} \right) = (2600 \text{ W}) \left( \frac{\cos 25^\circ}{\cos 65^\circ} \right) = 5600 \text{ W}
\]
62. **REASONING AND SOLUTION** The polarizer will transmit a maximum intensity of 
\( \frac{1}{2} \overline{S}_U + \overline{S}_p \), when its axis is parallel to the polarization direction of the polarized component of the incident light. Then the light intensity at the photocell is

\[
\overline{S}_{\text{max}} = \left( \frac{1}{2} \overline{S}_U + \overline{S}_p \right) \cos^2 \theta
\]

The polarizer transmits minimum light intensity of \( \frac{1}{2} \overline{S}_U \) when its axis is perpendicular to the polarization direction of the polarized incident light, so

\[
\overline{S}_{\text{min}} = \frac{1}{2} \overline{S}_U \cos^2 \theta
\]

Solving Equation (2) for \( \overline{S}_U \) gives

\[
\overline{S}_U = \frac{2 \overline{S}_{\text{min}}}{\cos^2 \theta}
\]

Using Equation (3) in Equation (1) and solving give

\[
\overline{S}_p = \frac{\overline{S}_{\text{max}} - \overline{S}_{\text{min}}}{\cos^2 \theta}
\]

Using Equations (3) and (4), we find that the percent polarization is

\[
\frac{100 \overline{S}_p}{\overline{S}_p + \overline{S}_U} = \frac{100 \left( \overline{S}_{\text{max}} - \overline{S}_{\text{min}} \right)}{\overline{S}_{\text{max}} - \overline{S}_{\text{min}} + 2 \overline{S}_{\text{min}}} = \frac{100 \left( \overline{S}_{\text{max}} - \overline{S}_{\text{min}} \right)}{\overline{S}_{\text{max}} + \overline{S}_{\text{min}}}
\]